Interactive Computer-Aided Design of Control Systems*

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Abstract

We describe four computer packages for interactive computer-aided design of control systems. DIGICON (digital control) is a computer aid to the design of digital controls for single-input single-output (SISO) systems using the state-space techniques of pole and zero assignment. CONCON (continuous control) is a similar package for the design of SISO continuous systems using the same techniques. The program DOPTICON (discrete optimal control) is a computer aid for the design of discrete optimal control systems using the linear quadratic Gaussian (LQG) theory and OPTICON (optimal control) is its continuous-time counterpart. All four packages contain algorithms for the computation of poles and zeros of the resulting designs as well as programs for evaluation of their transient responses. In the sequel we describe only two of the packages, namely DIGICON and DOPTICON, as the other two packages have very similar descriptions.

DIGICON [1] assumes that the dynamic object to be controlled is the plant, described by the equations:

\[ \dot{x}(t) = Fx(t) + Gu(t - \lambda) \]
\[ y(t) = Hx(t) + Ju(t) \]

where

- \( x \) = state vector, \( N_x \) dimensions*
- \( u \) = control vector, \( N_u \) dimensions
- \( y \) = output, \( N_y \times 1 \)
- \( \lambda \) = time delay
- \( F \) = system matrix
- \( G \) = input matrix
- \( H \) = output matrix
- \( J \) = direct transmission matrix

Since we are interested in digital controls, we require a sampled-data or discrete model of the plant. Usually we will assume that \( u \) is piecewise constant as occurs with a controller acting via a zero order hold. In any event, the discrete evolution of (1) is given by

\[ x(k + 1) = \phi x(k) + \Gamma u(k) \]
\[ y(k) = H_d x(k) + J_d u(k) \]

*If \( \lambda = 0 \), \( x \) is not really the state, but rather the state has infinite dimensions in this case.

*Received July 20, 1981. Accepted by Associate Editor E. R. Barnes.

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where \( x, u, \) and \( y \) are the state, input and output, respectively, and

\[
\begin{align*}
\Phi &= \text{discrete system matrix} \\
\Gamma &= \text{discrete input matrix} \\
H_d &= \text{discrete output matrix} \\
J_d &= \text{discrete direct transmission matrix}
\end{align*}
\]

Notice that if \( \lambda > \sigma \), then the dimension \( x(k) \) and \( \Phi \) will be greater than \( N \), the dimension of \( x(t) \) and \( F \).

The control for the discrete plant is implemented by a discrete dynamical system called the controller described by the equations

\[
x_c(k + 1) = Ax_c(k) + By(k) + Mr(k) \\
u(k) = Cx_c(k) + Dy(k) + Nr(k)
\]

In (3), \( x_c \) is the controller state and \( r \) is the system reference input. The vectors \( y \) and \( u \) are the plant output and input as in (2). Also, we define

\[
\begin{align*}
A &= \text{controller system matrix} \\
B &= \text{controller input matrix} \\
C &= \text{controller output matrix} \\
D &= \text{controller direct-transmission matrix} \\
M &= \text{controller reference input matrix} \\
N &= \text{controller reference signal direct transmission matrix}
\end{align*}
\]

The controller described by (3) is the result of a control law, \( u = -Kx \) and an estimator which approximates the plant (discrete) state, \( x(k) \), by \( \hat{x}(k) \). The estimator, in turn, has an error equation.

\[
\hat{x}(k + 1) = [\Phi - LH_d] \hat{x}(k).
\]

In (4), the matrix \( L \) is the estimator gain matrix.

The control gain \( K \) is selected to make the closed loop poles of the plant be located at the roots of the characteristic polynomial \( \alpha_c(z) \). The estimator gain \( L \) is selected to make the characteristic roots of the error Eq (4) be located at the roots of the estimator characteristic polynomial \( \alpha_e(z) \). The reference-input matrices \( M \) and \( N \) are selected to give desired locations to the variable zeros in the system transfer function. These zeros may be arbitrarily chosen, may be chosen to guarantee that the error Eq (4) is independent of \( r \), or, finally, may be chosen so that the only system feedback is via system error, \( r - y \) [1].

The DOPTICON program provides computer aids to solve the steady-state discrete-time optimal control problems. The user specifies the matrices, the weighting matrices in the performance index, and the covariances of the noise sources. Depending on the options chosen, the optimal controller gains, filter gains, the RMS state and control responses are computed as well as the related eigensystems. DOPTICON assumes that the system under consideration is the constant coefficient discrete-time linear system of the form

\[
x(k + 1) = \Phi x(k) + \Gamma u(k) + \Gamma_1 w(k) \\
y(k) = H_d x(k) + v(k)
\]

where \( w \) and \( v \) are the process and sensor noises with covariance matrices \( R_w \) and \( R_v \) respectively. DOPTICON assumes a quadratic cost function and minimizes the following loss function in the statistical steady state.

\[
J = E \left[ \frac{1}{2} \sum_{k=0}^{\infty} x^T(k)Q_1 x(k) + u^T(k)Q_2 u(k) \right]
\]

where \( Q_1 \) and \( Q_2 \) are the symmetric non-negative definite state and control weighting matrices, respectively. The program uses the certainty-equivalence principle which states that the optimal feedback system is the optimal state feedback control law applied to an optimal estimate of the state.

It is known that the steady-state optimal control of (5) with respect to (6) with known states is given by

\[
u = -Kx
\]

where the matrix \( K \) is the control gain. \( K \) is computed from

\[
K = (Q_2 + \Gamma^T S \Gamma)^{-1} \Gamma^T S \Phi
\]

where \( S \) is the solution to the discrete algebraic Riccati equation and can be found from the solution of the control Hamiltonian by eigenvector decomposition. If \( x \) is not available but control must be based on the noisy observations, \( y \), then it is known that the optimal control is given by

\[
u = -K\hat{x}
\]

where \( x \) is the least square estimate of \( x \) given \( y \) and satisfies the corrected model (or Kalman filter) equation:

\[
\hat{x}(k) = \hat{x}(k) + L(y(k) - H_d \hat{x}(k)) \text{ observation update} \\
\hat{x}(k + 1) = \Phi \hat{x}(k) + \Gamma u(k) \text{ state update}
\]

In (10) the steady-state estimator gain \( L \) is given by a solution dual to that for \( K \) in (7). If we combine (5), (8) and (10), and define the state error \( e(k) \equiv \hat{x}(k) - x(k) \), we can write the solution in the form:

\[
x(k + 1) = (\Phi - \Gamma K)(x(k) - LH_d e(k) - v(k)) \\
x(k + 1) = (\Phi - \Gamma K)(x(k) - LH_d e(k) + L v(k))
\]

We define the following mean square quantities:

\[
\begin{align*}
P_m &\triangleq E \{ e(k)e^T(k) \} \\
R_x &\triangleq E \{ \hat{x}(k)\hat{x}^T(k) \} \\
R_x &\triangleq E \{ x(k)x^T(k) \} = R_x + P_m \\
P &\triangleq E \{ (\hat{x}(k) - x(k))(\hat{x}(k) - x(k))^T \} \\
R_x &\triangleq E \{ \hat{x}(k)\hat{x}^T(k) \} = R_x + P_m - P
\end{align*}
\]
$P_m$ is found from the solution of the filter Hamiltonian by eigenvector decomposition and then $P$ is computed as follows:

$$P = \Phi^{-1}(P_m - \Gamma_1 R_w \Gamma_1^T)\Phi^{-T}$$  \hspace{1cm} (13)$$

The estimator gain is given by:

$$L = P_m H_d T (H_d P_m H_d^T + R_v)^{-1}$$  \hspace{1cm} (14)$$

As an indication of the control effort required by a given design, we frequently compute the covariance of the control. When $e = 0$ (perfect state measurement) this is

$$E\{uu^T\} = KR_x K^T$$  \hspace{1cm} (15)$$

The state covariance $R_x$ is the solution to the discrete Lyapunov equation:

$$R_x = \Phi_c R_x \Phi_c^T + \Gamma_1 R_w \Gamma_1^T$$

(16)

$$\Phi_c = \Phi - \Gamma K$$

When an estimator is required, the control covariance is given by

$$E\{uu^T\} = KR_x K^T$$  \hspace{1cm} (17)$$

and $R_x$ is the solution to

$$R_x = \Phi_c R_x \Phi_c^T + (P_m - P)$$  \hspace{1cm} (18)$$

The square roots of the diagonal terms in (15) and (17) are referred to as the "control RMS response" and those of $R_x$ as the "state RMS response".

DOPTICON solves for $K$ and $L$ by the eigenvector decomposition of the associated Hamiltonians. The eigensystems of the open loop and closed loop systems are provided as well as the RMS state and control responses. Programs are provided for the computation of the zeros of the resulting designs and the evaluation of impulse and step transient responses.

II. Language

All four packages have been written in APL [5]–[7].

III. Typical Application

The programs described herein can be used in the design of control systems for electromechanical servomechanisms, process control systems and the design of aircraft autopilots to name a few. In a typical use of these packages, the user is prompted for input data either in the form of matrices or data elements. The design then proceeds by a call to the proper functions depending on the design in mind. Once the design has been completed, the programs determine the overall system matrices. The user can then look at the plots of the transient responses and compute the zeros of the system. Since the programs are interactive, iterations on the preliminary design can be carried out quickly and efficiently.

IV. Limitations

These programs have been written for systems up to order 10 to 20. DOPTICON assumes that the plant does not have any pure delays and the cost on the control is nonsingular. If this is not the case, the problem can be formulated and solved as the solution to a generalized eigenvalue problem [8].

V. Availability

A user’s manual containing the description, source code and an illustrative example is available at reproduction cost from the authors. The documents are as follows:

1. DIGICON: Interactive Design of Digital Controls
2. CONCON: Interactive Design of Continuous Controls
3. DOPTICON: An APL Workspace for the Interactive Design of Digital Optimal Controls
4. OPTICON: Interactive Design of Continuous Optimal Controls

The programs have been tested on the IBM 370/3033 under VS APL at Stanford University and have performed well for various examples.

VI. Example

We present a simple example of the use of DIGICON for a double integrator plant. The data are entered using the function INPUT as follows.

```
INPUT
TO INPUT MATRICES, TYPE 1
TO INPUT DATA ELEMENTS, TYPE 2
U:
2
HOW MANY STATES?
U:
2
HOW MANY CONTROLS?
U:
1
HOW MANY OUTPUTS?
U:
1
ROWS OF P
ROW 1 = ?
U:
1
ROW 2 = ?
U:
1
0 1
0 0
COLUMNS OF Q
ELEMENTS OF COLUMN 1 = ?
U:
0 1
0
ROWS OF H
ROW 1 = ?
U:
1
1 0
ROWS OF J
ROW 1 = ?
U:
0
0
```

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The discrete model parameters are found using the function \texttt{SAMPLE} \( \phi \) with the sampling period \( T = 0.5 \) sec.

\begin{verbatim}
SAMPLE
TO USE \& AND \texttt{C}, TYPE 1
TO INPUT NEW SYSTEM, TYPE 2
\end{verbatim}

\begin{verbatim}
  1
SAMPLE PERIOD =
\end{verbatim}

\begin{verbatim}
  .5
SYSTEM DELAY \texttt{LAMBDA} =
\end{verbatim}

\begin{verbatim}
\phi
  1 0.5
  0 1
\end{verbatim}

\begin{verbatim}
\gamma
  0.125
  0.5
\end{verbatim}

\begin{verbatim}
\texttt{HD}
  1 0
\end{verbatim}

\begin{verbatim}
\texttt{JD}
  0
\end{verbatim}

This system has a zero at \(-1\). Using the function \texttt{CONLAW}, the controller poles are placed inside the unit circle at radius 0.6 and an angle of 25° (corresponding to \( \xi = 0.7 \)).

\begin{verbatim}
CONLAW
TO USE \phi, \gamma, \texttt{HD,JD}, TYPE 1
TO INPUT OTHER MATRICES, TYPE 2
\end{verbatim}

\begin{verbatim}
  1
TYPE 2 ROOTS AS RADIUS,ANGLE(DEGREES)
ROOT 1
\end{verbatim}

\begin{verbatim}
  .6 25
THE CONTROL GAIN IS \( K = \)
  1.089722622 1.552430656
CLOSED LOOP MATRIX IS \texttt{PHIC} =
  0.8637846722 0.3059461681
  0.5448613111 0.2237846722
\end{verbatim}

The function \texttt{REDU}EST is called to design a reduced order estimator with its pole placed at the origin. If we cancel the estimator pole, the controller matrices are computed as follows.

\begin{verbatim}
REDU
TO USE \phi, \gamma, \texttt{HD,JD}, TYPE 1
TO INPUT NEW DISCRETE MATRICES, TYPE 2
\end{verbatim}

\begin{verbatim}
  1
TYPE 1 ROOTS AS RADIUS,ANGLE(DEGREES)
ROOT 1
\end{verbatim}

\begin{verbatim}
  0 0
TO ASSIGN ZEROS, TYPE 1
TO USE ERROR CONTROL, TYPE 2
TO CANCEL ESTIMATOR POLES, TYPE 3
\end{verbatim}

\begin{verbatim}
SYSTEM MATRIX \( AO = \)
  0.3881076639
INPUT MATRIX \( BO = \)
  4.732811482
OUTPUT MATRIX \( CO = \)
  1
DIRECT MATRIX \( DO = \)
  4.194583333
REFERENCE INPUT MATRIX \( MO = \)
  0.422226552
REFERENCE DIRECT MATRIX \( NO = \)
  1.089722622
\end{verbatim}

The function \texttt{SYSMAT} is called to obtain the overall system matrices. The step response of the closed loop system is obtained using the function \texttt{STEP}. The system has a velocity error coefficient \( K_v = 0.7019 \).

\begin{verbatim}
SYSMAT
OVERALL SYSTEM \phi MATRIX, \texttt{SPHI} =
  0.4756770083 0.5
  2.097291967 1
OVERALL SYSTEM INPUT MATRIX \texttt{SGAMMA} =
  0.1362153278 0.5448613111
  0.422226552
OVERALL SYSTEM OUTPUT MATRIX \texttt{SH} =
  1 0
OVERALL SYSTEM DIRECT MATRIX \texttt{SJ} =
\end{verbatim}

\begin{verbatim}
STEP
TO USE \texttt{PHIC, GAMMA,HD,JD}, TYPE 1
TO USE \phi, \gamma, \texttt{HD,JD}, TYPE 2
TO INPUT DISCRETE SYSTEM, TYPE 3
\end{verbatim}

\begin{verbatim}
  2
PEAK VALUE = 1.024808108 5 SECS
MORE? TYPE THE NUMBER OF POINTS YOU WANT
\end{verbatim}

\begin{verbatim}
  1
\end{verbatim}

\begin{verbatim}
PEAK VALUE = 1.024808108 7.5 SECS
MORE? TYPE THE NUMBER OF POINTS YOU WANT
\end{verbatim}

\begin{verbatim}
  0
FOR A DIFFERENT PLOT SCALE SIZE TYPE 1
TO SEE A DIFFERENT OUTPUT FROM THIS STEP, TYPE 2
TO EXIT STEP, TYPE 3
\end{verbatim}

\begin{verbatim}
  3
\end{verbatim}

If we use error control instead (the second choice in \texttt{REDU}) and after a call to \texttt{SYSMAT}, the step response of the system would be as shown below.

\begin{verbatim}
STEP
TO USE \phi, \gamma, \texttt{HD,JD}, TYPE 1
TO USE \phi, \gamma, \texttt{HD,JD}, TYPE 2
TO INPUT DISCRETE SYSTEM, TYPE 3
\end{verbatim}

\begin{verbatim}
  2
\end{verbatim}

\begin{verbatim}
PEAK VALUE = 1.422226552 5 SECS
\end{verbatim}
Notice that the system has a much higher overshoot than before. We can increase $K_r$ by zero assignment (the first choice in REDUINST). For example, if we place a zero at 0.587544 and after a call to SYSMAT, the closed loop step response is as follows.

```
```

The velocity error coefficient $K_r = 1.4038$ and is double that of the first case.

As a check, the closed loop zeros and poles of the system are computed using the functions ZERO and EIGEN respectively.

```
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VII. Acknowledgments

We would like to thank Professor David Macneil for providing us with the APL functions for computing eigenvalues and eigenvectors and Dr. Gürcan Aral for contributing the transient response program. This work was supported in part by NASA under grant NGL 05-020-007 project 6303.

References

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Call for Computer Control Articles
IEEE Control Systems Magazine

One of the services the IEEE CSM wishes to initiate is the dissemination of information on computer software and hardware for use in control systems analysis and design. An initial set of guidelines for articles is proposed here. It is expected that these guidelines will evolve to suit the needs of the readers as the IEEE CSM matures.

I. Software Articles:

1. Articles should describe a general purpose program which will be of interest to a large audience of control system designers. The article should not contain the code itself.
2. The code should be available to the public in an easily transportable high level language such as FORTRAN, BASIC, APL, PASCAL, or PL/I.
3. The program must be well documented, both for the programmer and the user, and tested.
4. The code should be available at nominal cost, i.e., no profit should be made from publication of the article.

5. As a minimum, the article should include the following:
   a. An abstract
   b. The purpose of the program
   c. The language(s) used
   d. Typical applications
   e. Limitations
   f. Instructions for obtaining the code and documentation
   g. Specific computers and compilers on which the code has been tested
   h. An example

II. Hardware Articles:

1. Articles should address a specific problem on the hardware aspects of computer control systems. Particular attention is paid to microprocessor and microcomputer control systems.
2. The articles should include an actual implementation of the control action and/or specific hardware problem.
3. As a minimum, the article should include the following:
   a. An abstract
   b. The purpose of the computer control problem
   c. The computer or microcomputer used with corresponding codes, if any.
   d. A discussion on quantization errors, speed, etc.
   e. An explanation of the actual implementation of the hardware problem.

III. News Articles

1. News articles include reports on laboratory projects involving computer control or evaluation of components such as actuators, sensors, electric or hydraulic motors, etc. These articles are designed to acquaint readers with on-going laboratory experiments and research projects.
2. New articles will also include brief notes describing clear computational or programming tools that are useful in computer programming application.

IV. Submission Instructions:

Five (5) copies of the article should be submitted to:
Dr. E. R. Barnes, Associate Editor
IEEE Control Systems Magazine
IBM T. J. Watson Research Center
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