Control Characteristics of a Multifan Hovering Mechanism

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ABSTRACT: A multifan hovering mechanism is investigated as a vehicle for field work in a mountainous region. This paper presents the control characteristics of the hovering mechanism, including the effects of level and sloping ground. A stable controller is derived and experimental results are presented using a scale model. Work is being continued to improve the hovering characteristics.

Introduction

It is difficult to travel in rugged mountainous regions to do field work such as construction, forestry, or maintenance of power lines. At present, the only vehicle suitable for mountainous terrain is a helicopter. However, a helicopter is not suitable for work in close proximity to the ground because of large rotor and aerodynamic effects. In concept, a hovering mechanism should be appropriate for this type of task. However, a single-fan mechanism with a flexible skirt was tried by forestry researchers with little success.

The purpose of this paper is to examine the control characteristics of a multifan hovering mechanism. Similar hovering mechanisms with mobility and small size, e.g., VTOL aircraft, have been developed by several institutions, but they have not been put to practical use. Evidently, difficulties were encountered due to handling instability, and due to power inefficiency caused by the use of small propellers. However, recent improvements in computer technologies may solve handling instability, and extremely light engines under development in the automobile industry may improve power efficiency. These factors give improved prospects for practical use.

This paper presents analytic and experimental results using a scale-model vehicle with four fans. The scale model is driven by electrical motors controlled by changing rotational speed. The experimental control results are analyzed based on the assumption that the lift distribution varies as a function of the distance to the ground for both level and sloping ground. The scale model is intended to provide information that can be used to help develop a more sophisticated model with engine and pitch control.

Formulation of Mathematical Model

The structure of the scale model is illustrated in Fig. 1. Four DC motors are fixed on a cross-type frame. The distance d from each axis to the center is 25 cm. Every axis is tilted by a small angle e (3 degrees in our model) from the vertical z axis to produce a horizontal component Fz of the thrust F. Thus yaw control is made by the balance of the two moments. The rotational direction of each propeller is determined so that the directions of the reaction torque and the moment coincide. Since e is small, it is assumed that F = eT and the vertical component of T equals Tz.

Equations of Motion

Let I, J, and J be the vehicle moments of inertia, and N and N the roll and pitch torques to the th fan produced by ground effects. Because the vehicle motion is slow, air resistance is neglected. Gyro effects of the fans are also neglected. With these assumptions, we have the following equations for pitch, roll, and yaw motions:

\[ I_\phi = d(T_4 - T_2) + N_\phi \]  
\[ I_\theta = d(T_3 - T_1) + N_\theta \]  
\[ I_\psi = de(T_2 + T_4 - T_1 - T_3) \]

Formulation of Ground Effect

The thrust tends to increase as the vehicle approaches the ground with constant motor inputs. If D is the size of the vehicle and Tm is the thrust without ground effect, the thrust T at an average height h from the ground is approximated by the following equation for some constants k and h (1):

\[ T = T_m(1 + kD/(h_1 + h_0)) \]

Let S be the area of the rotational surface of the fan and h, the height of each point on the surface from the ground. Assuming that the thrust is distributed on the rotating surface of the fan and the density has the same form as Eq. (4), the torques N and N are determined by the following equations:

\[ N_m = (T_m/S) \int kD/\sqrt{h_1 + h_0} \, dS \]
\[ N_m = (T_m/S) \int kD/\sqrt{h_0} \, dS \]

Consideration of the Delay of Fan Rotation

When motor input u is changed, fan rotation is delayed because of inertia and air resistance. The delay can be estimated ex-

![Fig. 1. Scale-model structure.](image-url)
This graph shows a delay of 0.1 sec. experimentally. Figure be regarded as due to the change from the center of the model is constant, and $T_{in}$ is equal to $T_{in}/4$ for all $i$. If $L$ is the radius of the fan, Eq. (6) becomes

$$N_{in} = -kT_{in}/D(32)(L/h_i + h_{in})$$

(11)

The first term on the right side of Eq. (2) becomes

$$dT_{in}/dt = -(kT_{in}/D)h_0^3$$

(12)

The coefficient $(dT_{in}/dh_i)_{in}$ in Eq. (10) is evaluated graphically from experimental data.

For the particular case with the parameters determined from the scale model, solution of a cubic polynomial gives the predicted oscillation period as a function of the height $h$, and this predicted period can be compared with experimental results. For heights of 3 and 10 cm (for the scale model), the predicted periods are 0.69 and 1.05 sec, respectively. Experimental results shown in Fig. 3 agree with this prediction. Therefore, it can be concluded that the formulation is correct, at least for small deviations from an equilibrium state.

**Power, Thrust, and Height**

Figure 4 shows the configuration of the experimental devices. The model is connected to power supplies and computer interfaces through wires hung in the middle. The total weight is 1120 g. The parts of the wires that may influence the motion of the model are 40 g in weight and are neglected in the analysis. Three rate sensors are put at the center of the model to measure angular rates of pitch, roll, and yaw. A joystick and a dial are connected to the computer to pilot the model.

Relations among power $P$, thrust $T$, rotational speed $n$, and height $h$ were investigated experimentally. Power is measured electrically from the motor characteristics. $n$ is measured with a stroboscope. $T$ is estimated from the weight of the model.

The relation among $P$, $T$, and $n$ is expressed by the following well-known equations with coefficients $\alpha$ and $\beta$, which are functions of effective blade incidence.

$$P = \alpha n^3$$

(13)

$$T = \beta n^2$$

(14)

The effective blade incidence is defined as blade incidence minus induced flow angle. When blade incidence is fixed similar to the model presented herein, if $n$ increases, an induced flow angle increases, and $\alpha$ and $\beta$ decrease. Ground effect is known as a phenomenon that makes the induced flow angle decrease, $\alpha$ and $\beta$ increase, and $T/P$ increase.

First a single fan of radius 8.5 cm is used to measure the coefficient $\alpha$ for various heights. The results are shown in Fig. 5. From these curves, it is shown that the ground effect is small for $h < 8$ cm (in radius of the fan) and the curves become gentle for $n > 5000$ rpm.

Next, the performance of the four-fan model was measured. Figure 6 shows $h$ vs. $n$, where the dotted line is computed from the solid graph using relation (14) and assuming coefficient $\beta$ constant. From this graph and other results, the ground effect is non-negligible for $h < 20$, which corresponds to the size of the model.

Finally, power necessary for hovering is compared between $h = 0.5$ and $\infty$. From Figs. 5 and 6, $\alpha \approx 0.17$. $n = 5000$ for $h = 0.5$, and $\alpha \approx 0.12$, $n = 7200$ for $h = \infty$, so that the powers are about 85 W at $h = 0.5$ cm and 179 W at $h = \infty$. Thus the power needed to hover is reduced to one-half by the ground effect.

**Control with Ground Effect**

**Control System**

One of the difficult problems in the control of the hovering mechanism is the change of property with height. As was shown in the previous discussion, the oscillation period in the pitch motion changes with height. One approach is to design an adaptive controller, which adjusts the parameters to height. Since the property changes suddenly with the change of height in the ground effect, parameter adjustment without a height sensor seems difficult. Furthermore, the delay of thrust makes the controller unstable. A second ap-
approach—to design a robust controller—was adopted in this paper.

Other considerations are needed. First, a strong feedback produces side effects, such as up-down motion caused by yaw control. It may also bring about a runaway failure [2]. This consideration implies that a limiter should be introduced to the control system. Second, to hover above sloping ground, power for each fan should be variable because of ground-effect variation and should be changed corresponding to height. However, since the limiter restricts the value of a control variable, the difference of thrust cannot be compensated. Therefore, compensation of the difference of the ground effect due to the slope is manually given as a joystick command in our system, while usually a pilot command is input to a control system as a desired value.

Consider a small deviation from a statically balanced state for pitch motion only, since other motions are similar. Let \( p \) be a constant and \( \omega \) be an angular frequency varying with height. Neglecting the effect of the delay caused by the change of height, the state equations are simplified as follows.

\[
\dot{\theta} + \omega^2 \dot{\theta} = p(T_3 - T_i) \quad (15)
\]
\[
\tau \dot{T}_i = -T_i + u \quad (16)
\]

Measurable variables are \( \dot{\theta}, \dot{\phi}, \) and \( \dot{\psi} \), and it is difficult to estimate \( \dot{\psi} \) due to noise added to angular velocity. Therefore, each angle and angular velocity was selected to be fed back.

Figure 7 shows the controller, where it is assumed that \( p \) is normalized by the appropriate dimensions of \( \dot{\theta} \) and \( u \) and \( b \) is the bound specified by the limiter. A control variable \( u_b \) is defined by

\[
u_b = u_3 - u_1 \quad (17)
\]

The switch was added primarily to prevent instability due to the delay in Eq. (16). The switching condition is for an appropriate \( c \) as follows.

\[
k_1 \dot{\theta} + k_2 \dot{\theta} \quad \text{if} \quad |\dot{\theta}| \leq c
\]
\[
b \operatorname{sgn}(\dot{\theta}) \quad \text{if} \quad |\dot{\theta}| > c \quad (18)
\]

The idea of this switch is based on the following consideration. If the system is in an oscillation mode, then the frequency is bounded by the maximum value of \( \dot{\theta} \) for a certain amplitude. Instability due to the de-
lay of thrust increases as the period becomes shorter compared with the delay time. Therefore, if a bound is put on $\dot{\theta}$, i.e., the amplitude can be reduced to a certain value. This bound is also independent of the parameter $\omega$. Robustness can be obtained for the same reason as variable-structure systems [3].

Figure 8 shows the hardware implementation of the controller. All the analog signals are digitized by 8-bit AD converters. Motors are driven in PWM for a period of 12.8 ms, whose widths are specified in 128 levels by the computer. The sampling period of the controller is 10 ms. Data from rate sensors are sampled ten times and digitally low-pass filtered during each sampling period, where fan rotation generates noise. The control law of Eq. (18) is programmed in a minicomputer by replacing $\dot{\theta}$ with the sum of the time series of $\theta$. Resolution of the pitch and roll sensors is 0.78 deg/sec, and resolution of the yaw sensor is 0.23 deg/sec. The dial augments the control value equally to each fan by the setpoint to command the hovering height. Setpoint $X$ of the joystick determines the augmentation of a control value to either of the two fans on the X axis corresponding to the deviation from the middle point of the joystick. Setpoint $Y$ has the same effect on the fans on the Y axis. Thus, the joystick is used to compensate for the difference of the ground effect and also to move the model vehicle to the X and Y directions.

**Analysis by Simulation**

Stability is judged according to the trajectory on the $\theta-\dot{\theta}$ plane obtained by simulation. Equations to be considered are simplified by

$$\dot{\theta} + \omega^2 \theta = v$$  \hspace{1cm} (19)

$$\tau \dot{v} = -v + u$$  \hspace{1cm} (20)

Rough estimates of the variables and parameters are given by $\omega^2 < 150$ from Fig. 3, $\tau = 0.3$ from Fig. 2, and $b = 50$, $c = 0.3$, and $\theta < 0.2$ from experiment. Comparison will be made with a linear control law, where its feedback gains are determined assuming $r = 0$. Let the control law be

$$u = -k_1 \dot{\theta} - k_2 \theta$$  \hspace{1cm} (21)

Substituting Eq. (21) into Eq. (20), we have

$$\dot{\theta} + k_1 \dot{\theta} + (k_2 + \omega^2) \theta = 0$$  \hspace{1cm} (22)

Now, let us determine $k_1$ and $k_2$ for $\omega = 0$; that is, gains suitable for hovering at a high altitude. Then, $k_1$ and $k_2$ are determined for appropriate damping coefficient $\xi$ and the angular frequency $\omega_n$.

$$k_1 = 2 \xi \omega_n, \quad k_2 = \omega_n^2$$

Let $\xi = 1.2$ and $k_2 = 289$, for a value of $u$ close to the bound when $\theta = 0.2$. Then, $k_1 = 40.8$.

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**Fig. 8.** Hardware implementation of the controller.

**Fig. 9.** Trajectory of state on the $\theta-\dot{\theta}$ plane: (a) linear control law; (b) limiter used; and (c) limiter and switch used.

**Fig. 10.** Experimental data above level ground.
Figure 9 shows the results of simulation, where a thin line denotes the result for $\omega = 12$ and a thick line denotes $\omega = 0$. Part (a) is the result of the linear control law and shows instability due to the delay of response. If the delay is smaller, the motion becomes stable; if the gains are smaller, the motion becomes stable for $\omega = 0$. Part (b) shows the result when the limiter is added to the linear control loop. It is still unstable. Part (c) indicates the effect of the switch. To sum up the simulation results, the limiter makes the system slightly oscillative. The switch suppresses this oscillation, and also makes the system stable for both the change of $\omega$ and the delay of thrust.

**Experiments Above Level Ground**

Figure 10 shows experimental data at various heights. The results are analyzed below.

1. Small oscillation is observed, but divergent oscillations observed without control are suppressed.
2. The stationary deviation for $h = 10$ seems to be caused by the deflection of the slipstream by the ground.
3. Vertical motion is still observed, especially at a high altitude.
4. The motion is gentle at a high altitude because of the reduction of ground effect and the nature of the controller.
5. The model fluctuates slowly in the $x$ and $y$ directions while hovering, which implies low response time of the control system.

**Experiments Above Sloping Ground**

A new problem was considered before the experiments above sloping ground. Since the structure of the hovering mechanism is symmetric with respect to the $z$ axis, it is necessary to define the front side of the mechanism, which is heading toward the ascending direction of the slope. It was found by preliminary experiment that if the $x$ direction defined in Fig. 1 is the front, a yaw moment acts to enlarge the deviation of the yaw angle. Therefore, the front is chosen to be intermediate between the $x$ and $y$ axes.

Figure 11 shows examples of hovering. Problems found in the experiment are as follows:

1. There occurs little fluctuation in the $x$ and $y$ directions, while the model is apt to take one of the following actions: going up or down the slope, or along the contour line, while it faces toward the oblique direction. This fact implies that the structure in Fig. 1 is not necessarily suitable for slope. Therefore, an articulated mechanism is proposed so that front fans always face toward the ground, which was found to be stable above slope from a simple model.
2. Manual compensation is difficult, especially when the height is changing, as in Fig. 11(c), since the compensation and height should be changed simultaneously. Thus, an automated compensation is needed. However, the controller adopted in this paper is based on small disturbances, so a new control principle is required that allows a strong disturbance.

**Conclusions**

1. Effectiveness of the formulation developed in this paper was shown experimentally.
2. The switching element in the controller improved the performance of a linear controller and made hovering at various heights possible.
3. Experiments above sloping ground have revealed problems to be solved: Development of a controller that allows a strong disturbance, and development of a new mechanism suitable for slope. An articulated mechanism was proposed.

4. Gains and other parameters of the controller should be changed according to the height, although the parameter scheduling was not completed. A simple solution is to prepare two sets of parameters, with ground effect and without ground effect.

The author is currently developing a new scale model with an engine and pitch control of blades for better performance.

**Acknowledgments**

The author thanks the reviewers for their suggestions, which have improved this presentation. Thanks also to Dr. I. Fukui and Mr. K. Yamada for their encouragement of this work.

**References**


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October 1986