Model Reference Adaptive Control of a Direct-Drive DC Motor

Hans Butler, Ger Honderd, and Job van Amerongen

ABSTRACT: An adaptive time-optimal position controller for a direct-drive DC motor with the design based on the model reference adaptive approach is presented. The high-acceleration torque of new DC motors with permanent magnets permits a direct coupling of the load to the motor axis, avoiding the use of a transmission with its inherent disadvantages (such as backlash and friction). However, the direct coupling induces a large sensitivity of the motor behavior to load variations; therefore, a fixed, linear controller cannot provide an acceptable response under varying load conditions. Because of variations of load inertia, the desired response and the reference model in the adaptive controller have to be adjusted to the motor capabilities. This is achieved by estimating the load inertia by means of a least-squares method and adjusting the reference model accordingly. The controller is tested on a direct-drive motor, and the results are compared with those obtained with a fixed proportional-integral-derivative controller.

Introduction

Recently, DC motors that deliver a high torque on the motor axis have become available as a result of the development of new magnetic materials. The high-acceleration torque of these motors enables a direct coupling of the load to the motor axis, which obviates the necessity of using a transmission. Thus, the disadvantages of such a gear train (backlash, friction) can be avoided by using a direct-drive motor. Furthermore, the use of permanent magnets instead of conventional field windings makes this type of motor compact and its use attractive for robot applications.

However, the absence of a gear train involves a great sensitivity of the motor beha-

vior to variations in the load inertia. This can be seen by inspecting the following equation for the load acceleration, in which \( \omega_l \) is the load acceleration, \( N \) the gear ratio, \( M_m \) the motor torque, \( J_l \) the load inertia, and \( J_m \) the inertia of the motor axis.

\[
\omega_l = \frac{M_m}{(J_l + J_m N^2)}
\]

Increasing the gear ratio \( N \) decreases the load acceleration at a constant motor torque. Furthermore, the influence of load inertia \( J_l \) decreases in the denominator compared with \( J_m N^2 \); therefore, the sensitivity of load acceleration \( \omega_l \) to variations in \( J_l \) becomes smaller. For \( N = 1 \), the value of \( \omega_l \) depends heavily on \( J_l \). In a practical application, such as a robot, the load inertia may be unknown and can change during operation.

The aim of this paper is to design a time-optimal position controller for a direct-drive motor. With a step input, time-optimal control causes the motor, first, to accelerate maximally during a certain period of time, followed by a time span of maximum deceleration. The switching time must be such that at the end of the control the motor angle \( \theta_l \) equals the set point \( \theta_s \). Friction causes the maximum acceleration rate to differ from the maximum deceleration rate.

As a result of these conditions, the position controller must be robust with respect to changes in load inertia. Furthermore, the motor response should be comparable to a time-optimal response, which means that the motor current will be maximum during the largest part of the movement. These properties indicate that a linear controller with fixed parameters may not be satisfactory.

The paper describes the application of Model Reference Adaptive Control (MRAC) to a direct-drive DC motor. Because the maximum obtainable motor speed depends on the load inertia, the reference model cannot represent a fixed transfer function but must be made dependent on the actual speed capacity of the motor in order to achieve a time-optimal response. This is done by using a least-squares recursive estimator for the load inertia and using this estimation as a measure for the speed of the reference model.

The first part of this paper describes the model of a direct-drive motor and the design of a current loop as a way to implement the necessary current limitation. In the second part, MRAC is introduced as an extension of a proportional-integral-derivative (PID) controller. The estimation of the total inertia \( J_l = J_l + J_m \) is described, and the last part of the paper shows practical results when using the controller. A comparison of the MRAC approach and a PID controller is also presented.

Direct-Drive Motor Model

The transfer function of a standard model of a DC motor is shown in Fig. 1, where \( R_e \) is copper resistance, \( L_m \) the motor inductance, \( K_b \) the back electromotive-force (EMF) constant, \( J_l \) the total inertia \( (J_l + J_m) \), \( f \) the viscous damping, and \( M_c \) coulomb friction. In Fig. 1, it can be seen that the complete motor transfer consists of an electrical transfer function \( 1/(sl_m + R_e) \) and a mechanical transfer \( 1/(sl_l + f) \). Coulomb friction, which is caused by magnetic and mechanical hysteresis, results in a counteracting torque with magnitude \( M_c \) and a sign that depends on the sign of \( \omega_l \). This static model of the coulomb friction reflects only approximately the real friction effects [1] and is represented by the factor \( M_s \) \( \text{sign}(\omega_l) \). The coulomb friction introduces a steady-state error in the transfer from \( M_s \) to \( \omega_l \).

To protect the motor from overload, the manufacturer has specified constraints on the motor current. These limitations are three-fold:

1. Peak current limitation. The maximum occurring motor current is limited to avoid demagnetization of the permanent magnets. For the motor used, the peak current equals 10.4 A.

2. Continuous current limitation. To prevent the motor from overheating, the maximum constant current is limited to approximately half the peak current (4.5 A for the motor used). The continuous current limitation becomes active only after a certain period of time and can be disregarded in position control. However, if torque control is used, longer periods of maximum applied power can

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Thus, it properly, it on the motor current, although it the command input directly possible. For this reason, a current amplifier with transfer function to I,, (Fig. 2). When the current loop works the response.

There is a PI compensator $K_p (1 + \frac{1}{\tau_s})$ of the motor, this loop can be approximated by a transfer of 1. The current loop enables us to impose a specified current on the motor, and results in the simplified scheme of Fig. 3.

**Position Control of the Motor**

For position control of the angle $\theta_e$, position and velocity feedback are present. The motor angle $\theta_e$ is measured by a resolver mechanism. The Hybrid Resolver Control Transformer (HRCT) controlling the resolver has a 14-bit digital set-point input and an input for the resolver signal, and has an output sin($\theta_e - \theta_p$). The resolver principle is based on the fact that the voltage induced in a receiver coil depends on the angle between the transmitter and receiver coil. A 400-Hz sine wave is input to the transmitter and induces an AM modulated signal in the receiver, which is demodulated in the HRCT by applying asynchronous detection. This causes the 800-Hz disturbance signal in the HRCT output, which is filtered by a second-order network with transfer $\frac{1}{(\tau_f + 1)^2}$ ($\tau_f = 1.5$ msec). Its disturbance rejection at 800 Hz is 35 dB. An advantage of the resolver over, for example, an incremental encoder is its high resolution. However, the nonlinear sine function prohibits large set-point changes.

As well as a proportional factor $K_p$ and velocity feedback $K_v$, an integrating factor $K_i$ is implemented to avoid a steady-state error due to the presence of coloumb friction. To prevent the integrator from windup, the integrating action is active only during the linear part of the motion (i.e., when the absolute value of $L_i$ is less than or equal to $I_{lim}$). The full scheme of the motor, the current loop and the PID position controller, is depicted in Fig. 4. Because of the unknown inertia of the load, a fixed set of parameters $K_p$, $K_v$, and $K_i$ cannot satisfy the demands on motor behavior. If, for a certain load inertia $J_i$, the parameters provide a suitable response, a larger value of $J_i$ will result in too high an overshoot. A smaller load inertia, however, results in an overdamped, and so non-time-optimal, response. Furthermore, the magnitude of the step input affects the motor behavior due to the presence of current limiters.

Because of this, in earlier work, several nonlinear controllers have been designed:

1. A controller that estimates the load inertia and adjusts $K_i$ according to an adaptive law [2].
2. A controller with nonlinear velocity feedback using a polynomial function of $\omega_p$ [2].
3. A bang-bang controller that switches to linear control in the neighborhood of the end point $\theta_e$.

Here, MRAC has been applied to the motor system. In MRAC, the desired response of the controlled system is specified in terms of a reference model. During operation, the output or state vector of the process to be controlled is compared continuously to the output or state vector of the reference model. The obtained difference is used in an adaptive mechanism, which adjusts the parameters in the primary controller in such a way that the process response becomes equal to that of the reference model. A process being controlled by a suitable controller of which the parameters are properly adjusted by a Model Reference Adaptive System always exhibits the same behavior, independent of varying or unknown process parameters.

Applying MRAC to the DC motor, it was found that adjustment of only $K_i$ is sufficient to be able to impose an approximate time-optimal response. The controller is shown in Fig. 4 as an extension of the PID controller. Deriving the adaptive law using hyperstabil-
Assuming for simplification that the starting angle \( \theta_0 = 0 \), evaluating the preceding equations shows that the input signal must be switched when

\[
\theta = \theta_r(-I_\text{+}/(I_\text{-} - I_\text{+})) \quad (5)
\]

Note that if the coulomb friction is neglected, \( I_\text{-} \) equals \( -I_\text{+} \), and so the switching point equals \( \theta = \theta_r/2 \).

In the reference model, an estimation of the coulomb friction is used and \( I_\text{+} = 0.883 \, \text{A} \) and \( I_\text{-} = -1.117 \, \text{A} \). The speed of the reference model, determined by \( \omega_r \), must be equal to the speed that can be obtained by the motor: The current limiters make it useless to try to impose a greater speed on the motor than it can handle. Unfortunately, the total inertia, \( J_\text{t} \), is unknown and must be determined from measurement information. Here a least-squares recursive estimator is used to estimate \( J_\text{t} \). For this estimation, \( \dot{\theta} \) is considered as the transfer from the motor torque \( M_\text{t} \) [which equals \( K_r I_\text{+} - M_\text{t} \text{sign}(\omega_r) \)] to the angular acceleration \( \alpha_\text{p} \). This acceleration is calculated from the velocity \( \omega_r \) by a difference operator \( (1 - z^{-1})/T \). Because both \( M_\text{t} \) and \( \alpha_\text{p} \) are known signals, \( \dot{\theta} \) can be estimated by the following least-squares algorithm:

\[
(K_r/J_\text{t}) \left\{ \int_0^T I_\text{+} \, dt + \sum_{n=1}^N I_\text{-} \, dt \right\} = \theta_r - \dot{\theta}_0 \quad (6)
\]

The estimate of \( J_\text{t} \) is \( J_\text{t} = K_r/J_\text{t} \). In Eq. (6), \( \lambda \) is the forgetting factor and has a value of 0.95 during the experiments.
Although the calculated acceleration $\alpha_\theta = \left[\omega_\theta(k) - \omega_\theta(k - 1)\right]/T$ appears to be a noisy signal, no extra filter is needed because the least-squares algorithm by itself has sufficient noise-filtering properties. To avoid drift in the estimated parameter, the above-described identification procedure is switched off if the motor current $I_\theta$ drops below a pre-specified value. This is necessary because the excitation of the least-squares algorithm is insufficient if $I_\theta$ is small.

Experiments have shown that the described estimation scheme performs well and the resulting reference model response differs only slightly from the actual time-optimal response. The remaining difference is caused by the initial incorrectness of the estimate, which can be overcome by resetting the reference model to the actual velocity of the motor axis at a moment at which the estimate has reached a certain degree of accuracy. However, this procedure appears not to improve system behavior and is not used in the actual system.

The complete control scheme incorporates two-sided adaptation. A least-squares algorithm estimates the load inertia and adjusts the reference model to the motor capabilities. Because the motor response is made to approach the reference model response, an accurate estimation of the load inertia is essential. The reference model then represents a time-optimal response of the motor, and the adaptive laws in the MRAC scheme try to impose this behavior on the motor by adjusting $K_\theta$. In the next section, some results using this controller are discussed.

**Experimental Results**

The controller described earlier was tested on the type QT6205C motor from Inland with the following motor parameters (see Fig. 1): $R_m = 6.67 \ \Omega$, $L_m = 1.4 \times 10^{-3} \ \text{H}$, $K_l = 3.26 \ \text{Nm/rad}$, $K_v = 3.26 \ \text{V/rad} \cdot \text{sec}^{-1}$, $J_m = 0.022 \ \text{Nm} \cdot \text{sec}^2$, $J = 0.013 \ \text{Nm/rad} \cdot \text{sec}^{-1}$, $M_a = 0.38 \ \text{Nm}$, and $P_{\text{max}}$ (maximum power) = 627 W. The algorithms were implemented in FORTRAN on a MicroVAX II minicomputer, and the complete system is controlled by the real-time shell MUSIC (MULTipurpose Simulation and Control) package. The hardware used for communication with the motor control unit does not allow sample times smaller than 10 msec, which is large compared to the time constants of the motor. For this reason, the peak current has been lowered to 1 A in order to reduce the motor speed.

To the motor axis, an aluminum rail has been fixed, to which metal weights can be fastened in different positions. This allows $J_\theta$ to be varied from 0.117 to 0.460 Nm $\cdot$ sec$^2$. The proportional factor in the fixed controller is set to $K_p = 20$. To compare the PID controller and the MRAC scheme, $K_i$ and $K_\theta$ in the PID controller have been optimized for a step input of $\pi/8$ rad and a load inertia of 0.21 Nm $\cdot$ sec$^2$. For this optimization, the following criterion has been minimized:

$$\text{CRIT} = \int_0^T \left[ (\theta_p - \theta_\omega)^2 + \mu (\omega_p - \omega_\omega)^2 \right] dt, \quad \mu = 1$$

In this formula, $\theta_\omega$ and $\omega_\omega$ represent the outputs of a time-optimal motor model. The optimization yields $K_i = 2.4$ and $K_\theta = 20$.

In order to test the algorithm, step inputs of $\pi/8$, $\pi/4$, and $\pi/2$ rad, respectively, are applied, in combination with load inertias of 0.117, 0.210, and 0.460 Nm $\cdot$ sec$^2$. For each combination, the preceding criterion is evaluated for both the PID and the MRAC controller. The results are listed in the Table. If $\theta_\omega = \pi/4$ and $J_\theta = 0.21$, the criterion value for both the PID and MRAC controller is small, due to the fact that the PID controller is optimized for this combination. The responses in this case are similar. However,
if the load inertia is changed, the performance of the PID controller quickly deteriorates, whereas the performance of the MRAC controller remains good. A change in the magnitude of the step input has the same effect.

Obviously, the MRAC controller can improve the motor behavior considerably in the scope of this criterion. As shown in Fig. 6, where \( \theta_0 = \pi/8 \) rad and \( J_1 = 0.117 \text{ Nm} \cdot \text{sec}^2 \), the PID-controlled motor is clearly slower than the MRAC-controlled system. Under extreme conditions, the difference is even more apparent; Fig. 7 shows the motor behavior if \( \theta_0 = \pi/2 \) and \( J_1 = 0.460 \text{ Nm} \cdot \text{sec}^2 \). The MRAC scheme prevents the motor response from large overshoot.

**Conclusions**

Application of Model Reference Adaptive Control to a real direct-drive DC motor has proved successful and confirmed simulation results. The controller guarantees approximate time-optimal behavior of the motor if a step input is applied, independent of the load inertia and the magnitude of the step input.

Because of the limitations of the used computer configuration, the motor speed has been reduced by limiting the peak current to 1 A. If the maximum current is set to 10 A, a faster computer system should be used (e.g., a dedicated signal processor), or the system can be implemented in analog circuitry. At these high speeds, the estimation of the load inertia also must be faster. At present, research is being done on using a piezoelectric accelerometer to measure the angular acceleration \( \omega_a \), and analog electronics is being developed to calculate \( J_1 \) from \( I_1 \) and \( \omega_a \).

**References**


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**Fig. 7 Response with \( \theta_0 = \pi/2 \) rad and \( J_1 = 0.46 \text{ Nm} \cdot \text{sec}^2 \):** (a) using PID; (b) using MRAC.

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