Design of Turning Control for a Tracked Vehicle

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ABSTRACT: This article treats the design of a turning control system for an M113 tracked vehicle, which is modeled as a two-input/two-output system. Since the vehicle model is nonlinear, time varying, and implicit, it is replaced by a set of linear time-invariant models, which is valid over certain operating regions. The two inputs to the vehicle are throttle and steering. The two measured output variables are the vehicle's longitudinal speed and heading rate. Using the Quantitative Feedback Theory (QFT), a robust control system is obtained. The control loop for vehicle speed is designed to track the speed command and reject the disturbance caused by steering. The control loop for the vehicle heading rate is designed to track only the rate command, since the disturbance caused by throttle is negligible. The control system design is tested by simulation and shows satisfactory results.

Introduction

A previous article [1] described an automatic pilot that could drive an M113 armored personnel carrier as an autonomous land vehicle. Field tests have demonstrated the successful execution of road-following at speeds up to 20 km/h. However, a more sophisticated vehicle control system is required to respond faster and more accurately to a variety of commands including cruising, turning, and advancing a specified distance.

The design of the turning control system for a tracked vehicle is difficult because of complexities and uncertainties in the model. In particular, in this model, some nonlinearities lack explicit expression and many relationships are determined from empirical formulas. Since the internal models are so complex, it seems reasonable to consider external models or the system input-output relationships for the control design. The problems are how to obtain a simple and sufficient external model and how to design a controller based on such a model. Quantitative Feedback Theory [2]-[4] provides a systematic procedure for solving these problems.

According to the Quantitative Feedback Theory, the plant is treated as a black box. Only input-output behavior is considered. A set of linear time-invariant models, which exhibits the same input-output behavior over certain operating regions as the given nonlinear system, is regarded as equivalent to that nonlinear system. A robust controller is constructed based on the set of linear time-invariant models.

However, the design of controllers for real systems involving significant nonlinearities and parameter uncertainties is not that straightforward, especially with multi-input multi-output systems. This article treats the design of a turning controller for a vehicle model with two inputs and two outputs based upon QFT. A speed controller for a single-input/single-output vehicle model has been reported earlier [5].

The whole design procedure can be divided into four steps. First, the two-input/two-output nonlinear model is replaced by a set of two-by-two linear transfer function matrices. This set approximates the original nonlinear model in a confined operating region, because each linear transfer function in the set is derived from an input-output pair of the original nonlinear model. The nonlinearities of the original nonlinear model have been replaced by the parameter uncertainties of linear models. Second, this two-input/two-output linear control problem is converted into two single-loop design problems, which take into account both tracking and disturbance rejection. Third, the single loops are designed using the Quantitative Feedback Theory techniques. Finally, the controller is verified by simulation.

Conversion to Linear Model

The vehicle system block diagram shown in Fig. 1 consists of three main parts, which are the power train, the rigid body, and the ground interaction. The behavior of the power train is approximated by using a first-order differential system with a number of nonlinear empirical formulas. The rigid body is described by a five-degree-of-freedom nonlinear differential system. The ground interaction is modeled using Coulomb's friction law, and the complex interaction between the tracks and the ground leads to an implicit description. The control inputs to the vehicle are the throttle, the steering, and the brake. The vehicle outputs are the longitudinal speed, the transversal speed, heading rate, and left and right track speeds.

In our design, the measured outputs are the longitudinal speed y1 and the heading rate y2, and the inputs to the vehicle are the throttle u1 and the steering u2. The closed-loop step responses for the longitudinal speed and the heading rate are required to be within specified bounds. Under normal operating conditions, the longitudinal speeds vary between 3.5 and 8 m/sec, and the heading rates vary between 0.1 and 1 rad/sec. The linear models are derived from input-output pairs of the nonlinear model. Each output of the pair is within the specified bounds. Since the system lacks an explicit description, the desired input-output pairs cannot be generated by inverting the system equations. Instead, various combinations of the throttle, steering, and turning times are tried in order to generate a family of satisfactory input-output pairs. From experience, the throttle u1 and the steering u2 are assumed to be in the following form where r is the time to start turning:

\[
\begin{align*}
    u_1(t) &= v_1 + (1 - v_1) \exp(-\alpha_t), \\
    &\quad t \geq 0 \\
    u_2(t) &= 0, \quad t < r \\
    &= v_2 \left[ 1 - \exp \left( -\beta (t - r) \right) \right], \quad t \geq r
\end{align*}
\]

Different input-output pairs of the nonlinear model are generated with various combinations of \(v_1, v_2, \alpha, \beta, \) and \(r\). System identification techniques are used to find a two-by-two transfer function matrix for each input-output pair. A first-order form is assumed for each of the four elements of the transfer function matrix \(P(s)\), so that \(p_i(s) = a_i(s + b_j), \ (i, j = 1, 2)\). Since the throttle input does not affect the heading rate, one of the four elements of the transfer function matrix is zero—i.e., \(p_2(s) = 0\). Sam-
The inverse of the plant $F$ because the inverse of the matrix gain matrices. The transfer function models are given in the Table.

The closed-loop control configuration of the linear model $F$ is shown in Fig. 2, where $F$ and $G$ are diagonal two-by-two compensator gain matrices. The transfer function matrix $H$ of the closed-loop system from input $r$ to output $y$ is determined directly.

$$H = (I + PG)^{-1}PG$$

The inverse of the plant $P$ is particularly simple because $p_{22}$ is zero.

$$P^{-1} = \begin{bmatrix} \frac{1}{p_{11}} - \frac{p_{12}}{p_{11}p_{22}} & 0 \\ 0 & \frac{1}{p_{22}} \end{bmatrix}$$

Because the matrices $G$ and $F$ are diagonal, their product is diagonal. Hence, after taking the inverse of $(P^{-1} + G)$, the components $h_{ij}$ of $H$ can be determined directly.

$$h_{11} = p_{11}g_{11}f_{11}/(1 + p_{11}g_{11})$$
$$h_{22} = p_{22}g_{22}f_{22}/(1 + p_{22}g_{22})$$
$$h_{21} = 0$$
$$h_{22} = p_{12}d_{12}/(1 + p_{11}g_{11})$$

where

$$d_{12} = h_{22}p_{12}/(p_{11}p_{22})$$

The quantity $d_{12}$ corresponds to the disturbance on $y_2$ caused by input $r_2$, as shown in the block diagrams in Fig. 3.

Conversion to Single Loops

According to QFT, the design specifications are given by the time-domain step response of the closed-loop system. For example, the output $y_2$ due to a step input $r_2$ is required to be within specified bounds. This time-domain specification is then converted into a frequency-domain specification for the transfer function $h_{11}$ by the following procedure: The desired closed-loop step response is assumed to be that of a second-order system. We consider $h_{11}(s)$ in the following two forms:

$$h_{11}(s) = \omega_n^2/(s^2 + 2\xi\omega_ns + \omega_n^2)$$

where

$$\omega_n, \xi, z$$

We choose different combinations of $\omega_n$, $\xi$, and $z$ so that the step responses of these systems fall within the bounds of the time-domain specifications. The corresponding frequency-domain values are plotted on a Bode diagram; their envelope forms the frequency-domain specification for $h_{11}(s)$, as shown in Fig. 4. Similar bounds are derived for $h_{12}(s)$ and $h_{22}(s)$.

**Design Procedure**

There are many published papers on Quantitative Feedback Theory [2]-[4]. The compensator design for $h_{12}$ (the heading rate loop) is a single-input/single-output design problem. The compensator design for $h_{11}$ and $h_{22}$ (the longitudinal speed loop) is a little more complex since the compensator $g_1$ must be constructed to satisfy both the tracking and the disturbance rejection requirements. In either design, the main idea is as follows: Consider the set of linear transfer functions $P_i(s)$ describing an uncertain single-input and single-output system. The closed-loop system transfer function is

$$H_i(s) = \frac{P_i(s)G(s)F(s)}{1 + P_i(s)G(s)}$$

According to the frequency-domain specifications, the closed-loop transfer function is required to be inside the bounds $\alpha(\omega)$ and $\beta(\omega)$ at a set of specified frequencies $\omega_1, \omega_2, \ldots, \omega_n$, i.e.,

$$\beta(\omega) < 20 \log |H_i(\omega)| < \alpha(\omega), \quad \text{for} \ i \in \{1, \ldots, n\}$$

This implies that

$$20 \log |H_i(\omega_k)| - 20 \log |H_i(\omega_i)| \leq \alpha(\omega) - \beta(\omega), \quad \text{for all} \ i, k$$

An equivalent expression is

$$\max_{\omega(i,k)} 20 \log N_i(\omega) = 20 \log N_i(\omega)$$

where

$$N_i(\omega) = |P_i(\omega)Me^{i\phi}| + |1 + P_i(\omega)Me^{i\phi}|$$

$$M = |G(\omega)|$$

$$\phi = \angle G(\omega)$$

Notice that we have cancelled the term $F(\omega)$. If we assume that $P_i(\omega) \neq 0$ for all $i$ and $\omega$, then for any $\omega$ and $\phi$, we can always find a large enough $M$ to make both log functions close to zero. Hence, the above inequality is satisfied. Let $B(\omega, \phi)$ be a lower bound on such $M$. For a set of frequencies
the set of numbers $|P_i(j\omega) G(j\omega)|/|1 + P_i(j\omega) G(j\omega)|$, $i = 0, 1, \ldots, n$, close to each other in order to satisfy (2).

Lastly, we need to design a feedforward compensator $F(s)$ to bring the whole set of numbers within the bounds $\alpha(\omega)$ and $\beta(\omega)$, $\omega \in \{\omega_0, \omega_1, \ldots, \omega_m\}$, in order to satisfy (1).

The above description only sketches the basic concepts in QFT. Interested readers should consult many articles on this subject for further details [2]-[4]. In our case, the compensators obtained are as follows:

$$f_i(s) = \frac{s + 1}{0.5(s + 1)(s + 1)}$$

$$g_i(s) = \frac{0.43(s + 1)}{(0.05 + 1)(0.2 + 1)}$$

$$f_2(s) = \frac{s + 1}{5(s^2 + 14s + 4 + 1)}$$

$$g_2(s) = \frac{-0.3579(s + 1)(s + 1)(s + 1)(s + 1)}{s(s + 1)(s^2 + 14s + 4 + 1)}$$

Simulations

The last step is to perform the closed-loop simulations. The discretized versions of $G(s)$ and $F(s)$ at a 10-Hz sampling rate are used with the nonlinear vehicle model in the simulations. Limiters are also included for the inputs to the vehicle model. Figure 5 shows one typical result of the closed-loop system simulation. The response time and overshoot satisfy the specifications except for some steady-state errors in the tracking of speed.

In order to eliminate the steady-state error in speed, we add an integrator to $g_i$. The integrator is used only when the tracking error is small. In this way, we can both eliminate the steady-state error and improve the dynamic response. The switching formula and the integrator gain are selected through simulations.

Furthermore, different speed commands such as sinusoidal functions are considered. To track such speed commands, a brake input of constant value is applied whenever the speed exceeds the command value. Further details of the design can be found in [6].

Discussions

This article has investigated the application of the Quantitative Feedback Theory to the design of a robust turning controller for
the tracked vehicle M113. Since we are dealing with a real-world design problem, certain engineering sense and judgment are needed.

For example, one needs to choose the appropriate set of input-output pairs in order to derive the set of linear models to replace the nonlinear model. The most crucial step in the design is the loop shaping process, which requires some designer experience. Hence, it would be very useful to derive efficient algorithms for computer implementation of this design approach.

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References


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