A Control Theoretic Model of Driver Steering Behavior

R. A. Hess and A. Modjtahedzadeh

Following well established feedback control design principles, a control theoretic model of driver steering behavior is presented. While accounting for the inherent manual control limitations of the human, the compensation dynamics of the driver are chosen to produce a stable, robust, closed-loop driver/vehicle system with a bandwidth commensurate with the demands of the driving task being analyzed. A technique for selecting driver model parameters is a natural by-product of the control theoretic modeling approach. Experimental verification shows the ability of the model to produce driver/vehicle responses similar to those obtained in a simulated lane-keeping driving task on a curving road. A technique for selecting driver model parameters is a natural by-product of the control theoretic modeling approach. Experimental verification shows the ability of the model to produce driver/vehicle responses similar to those obtained in a simulated lane-keeping driving task on a curving road.

Control Technology for Automotive Engineering

Active control technology, which is now routinely used in modern high-performance aircraft, is finding its way into the realm of automotive engineering [1].

The design and development of systems for four-wheel steering, active suspensions, active, independent braking and "drive-by-wire" steering provide the engineer with considerably more freedom in altering vehicle handling qualities than existed in the past. Mathematical models of driver steering behavior can serve as useful tools in analytical investigations of proposed vehicle control systems provided that they meet two criteria. First, they should be able to predict driver/vehicle responses accurately enough to allow valid engineering decisions to be made concerning the acceptability of proposed vehicle control schemes, and second they should be relatively easy to use by an engineer who may not be a manual control specialist, i.e., they should require a minimum of "art" in their application.

To be sure, the subject of mathematically modeling driver steering behavior is not new. A sampling of research in the area, dating from 1961 can be found in [2-12]. It is probably safe to say that none of the models presented in these references meet both criteria stated above. This is not intended as a criticism. Most, if not all, of the research described was directed at developing descriptive tools for the researcher rather than predictive tools for the practicing engineer. The work described here places increased emphasis on the latter role, developing predictive tools for driver steering models.

Control Theoretic Framework

Consider Fig. 1, which shows a driver/vehicle system in a lane-keeping task on a curving roadway. Assume that the vehicle is under "cruise control" and is maintaining some desired speed $u_0$. The driving task can be summarized as one of maintaining the lateral path error $y_R(t)$ near zero. This task suggests...
Proposed Driver/Vehicle Model

Vehicle Model

A brief discussion of the vehicle model is in order before the driver model can be introduced. For purposes of exposition, the description of the vehicle model will be as simple as possible.

Referring to Fig. 1, a linear, four-variable state space model was derived, with state variables $x(t)$, $r(t)$, $\psi(t)$ and $y(t)$, defining the component of vehicle velocity in $y_b$, the $y$ body-axis, yaw rate, heading angle and lateral deviation, respectively. No roll or suspension dynamics were considered in the model although their inclusion would pose no difficulties except added vehicle model complexity. The characteristics of the particular vehicle being modeled were obtained from a study

![Diagram of Driver/Vehicle Model](image-url)

Fig. 4. The driver/vehicle model.
summarized in [9] and represent those of a full-size car traveling at 50 km/h. For the purposes of this study, the vehicle dynamics are summarized by the following linear transfer function:

$$\frac{\delta_v}{\delta_{sw}} = \frac{7.69(s^2 + 2(0.372)(5.96) + 5.36^2)}{s(s + 4.44)(s + 5.33)}$$

m/h-rad. \hspace{1cm} (1)

**The Driver Model**

Fig. 4 is a block-diagram representation of the proposed driver/vehicle system, emphasizing the driver model elements. As the figure indicates, the driver model has been divided into high and low-frequency compensation elements, each of which are described in the following sections. As will be seen, this model will 1) allow the bandwidth and stability requirements of aggressive steering tasks to be met, 2) include a neuromuscular system mode, and 3) exhibit the desirable open-loop return ratio characteristics of Fig. 3.

**High Frequency Driver Compensation**

We begin by adopting a model which has been successfully employed in modeling human pilots in well-defined flight control tasks: the so-called "structural model" of the human operator [15]. This model constitutes the high-frequency driver compensation, where high-frequency refers to frequencies within an approximate one decade range around the crossover frequency of the overall driver/vehicle open-loop return ratio. Space does not permit a detailed discussion of the genesis of the structural model, and the interested reader is referred to [15].

The block labeled $G_{NM}$ is a simple second-order representation of the neuromuscular system of the driver's arms. Block $G_{n}$ has, as its input, the output of the neuromuscular system $\delta_{sw}$ which is the driver's steering input to the vehicle. Block $G_{pl}$ receives its input from the output of $G_{n}$. Both of these dynamic elements represent feedback of variables derived from the motion of human limbs and muscle tissue, and are referred to as "propioreceptive" feedback elements. The block $G_{d}$ is a time delay representing the inherent human signal processing delays. This delay is not equivalent to the $T_{d}$ mentioned previously, in that the latter delay includes the phase lag effects attributable to the remainder of the dynamics in the structural model, namely those emanating from the feedback loop involving $G_{NM}$ and $G_{n}$.

Although the high-frequency driver compensation involves an eight parameter model, experience in modeling the human operator in a variety of manual control tasks, e.g., [16], has shown that four of these parameters can be considered invariant, with the remaining four dependent only upon the human operator compensation (proportional, derivative, or integral) which will cause the $y_{n}/u$ transfer function in Fig. 4 to follow the dictates of the crossover model in the high-frequency region just defined. Table I summarizes these parameters. For the vehicle dynamics of (1), high frequency derivative compensation is required to cancel the vehicle pole at $s = -4.44$.
Note that the vehicle pole at $s = -5.33$ is essentially cancelled by the complex zero. This means that, in Table I, $k = 2$, and $T = 1/4.44\ \text{s}$, and the remaining model parameters are listed in the last row of the table.

Note that no feedback loop involving visually sensed lateral vehicle velocity has been employed in the model, despite the fact that we have formed the structural model based upon $y_v/k_w$ vehicle dynamics. The reason for avoiding a driver model with two feedback loops using visually (as opposed to proprioceptively) sensed variables has been pointed out previously. This is an important step in the modeling procedure. The high-frequency compensation in the driver model which is based on the $y_v/k_w$ transfer function, simply serves to tailor the high-frequency part of the open-loop return ratio. Note further that the feedback variables in the structural model are internal to the human and are assumed to involve proprioceptive organs such as muscle spindles, joint angle receptors, etc. [17].

**Low-Frequency Driver Compensation**

An examination of the transfer function $y_v/u$ in Fig. 4 at this point indicates that the desired open-loop return ratio characteristics of Fig. 3 can be obtained with the addition of the block $G_c$ in Fig. 4, where $G_c = K_c[1-(1/T_3)]$. For modeling purposes, the zero at $-1/T_3$ will be maintained a decade below the crossover frequency of the open-loop return ratio $y_v/e_k$. This decade separation represents a reasonable choice as it provides the desirable large, low-frequency magnitude shown in Fig. 3, without adversely affecting the gain and phase margins. Fig. 5 shows the resulting amplitude and phase characteristics of the open-loop return ratio when, for the sake of comparison with Fig. 3, a crossover frequency of 1 rad/s has been chosen. The amplitude peaking evident around 1011 rad/s is a result of the "proprioceptive" feedback loops around the neuromuscular system $G_{NM}$ in the structural model in Fig. 4. As will be seen, the maximum bandwidth achievable with the system of Fig. 5 is considerably greater than that possible with either of the two feedback possibilities discussed earlier.

With the linkage between $K_c$, $1/T_3$, and $\alpha_k$, we see that only one parameter in the driver/vehicle model is used to account for driver adaptation to different steering tasks with any set of vehicle dynamics. That parameter is the open-loop return ratio crossover frequency $\alpha_k$.

**Visual Guidance Cue**

A simple visual guidance cue, first proposed in [18], could be used by the driver in closing the outer feedback loop of Fig. 4. By realizing that $y_v(t) = u_o y_v(t)$ and, for typically configured vehicles $y_v(t) = u_o y_v(t)$ the variable $u$ in the model of Fig. 4 is equivalent to a weighted sum of heading and lateral displacement errors between the desired and actual vehicle paths. Thus, in the time domain, and referring to Eq. 6,

$$u(t) = K_c[y_v(t) - y_v(t)] + (1/T_3)[y_v(t) - y_v(t)] = K_c[y_v(t) + y_v(t)/u_o T_3] = K_c[y_v(t) + y_v(t)] = K_c[y_v(t) + y_v(t)] = K_c[y_v(t) + y_v(t)].$$

Thus, the variable $u$ is proportional to $y_v$, defined as the angle between the vehicle $X_B$ body axis and an "aim point" on the tangent to the lane centerline, a distance $u_o T_3$ ahead of the vehicle. The simple guidance cue defined in (2) is valid regardless of the driving task. Of course, a combination of high speed and low crossover frequency can lead to an aim point location far ahead of the vehicle. This simply means that $y_v(t)$ would be quite small as compared to $y_v(t)$, and the latter would become the primary visual cue in the driving task.

**Driving Simulator Data Comparison**

Driver/vehicle response data from a driving task summarized in [9] can now be used to evaluate the accuracy of the driver/vehicle model just described. The task used the vehicle dynamics summarized here by the lateral path to steering input transfer function of (1) with a constant speed $u_o = 50\ \text{km/h}$. The task consisted of lane-keeping on the curving roadway shown in Fig. 7. All the driver model
parameters associated with the high-frequency compensation have been chosen in the preceding section independently of the driving task. As just mentioned, $\omega_c$ (and consequently, $K_c$ and $1/T_c$) is the only driver/vehicle model parameter which will be varied to tune the model responses to those from the simulation experiment of [9]. This tuning procedure was accomplished by employing a simulation of the driver/vehicle system of Fig. 4 (including nonlinear roadway kinematics) and varying $\omega_c$ from run to run until the standard deviations of lateral deviation and heading errors of the driver/vehicle model were close to those obtained in the experiments of [9]. The resulting low-frequency compensation, $G_c(s)$, is given by:

$$G_c(s) = 1.75(s + 0.325).$$ (3)

The crossover frequency $\omega_c$ for the driver vehicle system for this $G_c(s)$ is 3.25 rad/s, with gain and phase margins of 5.4 dB and 24°, respectively. The corresponding closed-loop bandwidth defined as the lowest frequency for which

$$|\frac{\gamma_1(s)/\gamma_2(s)}{(s/\omega_c)}| = 3 \text{ dB}$$

was found to be 5.35 rad/s. The error standard deviation comparisons are:

<table>
<thead>
<tr>
<th>variable</th>
<th>experiment</th>
<th>model</th>
</tr>
</thead>
<tbody>
<tr>
<td>heading</td>
<td>0.50°</td>
<td>0.42°</td>
</tr>
<tr>
<td>lateral displacement</td>
<td>0.22 m</td>
<td>0.26 m</td>
</tr>
</tbody>
</table>

The statistics from the experiment were the result of eight simulation runs by each of six, well-trained test subjects. A comparison of low-frequency steering wheel inputs, i.e., those which correspond to the roadway curvature, indicated very close agreement between model and experiment. However, the experimental $\omega_c$ contained a low-amplitude high-frequency component which can be attributed to driver “remnant” [14]. The effect of such remnant injection is not considered significant in the evaluation of overall vehicle handling qualities, and hence, remnant was not included in the modeling effort here. Fig. 8 shows the steering input time histories for one of the drivers from the experiment and that from the model, with the former exhibiting the high-frequency remnant component.

It is interesting to note that the crossover frequency, gain and phase margins of the "tuned" driver/vehicle model are also representative of values which have been measured in single-loop manual tracking tasks in which the dynamics of the plant require high-frequency lead compensation on the part of the human similar to that required here [19]. This result suggests that, in the absence of driver simulation data with which to choose $\omega_c$, the wealth of theoretical and experimental information in the manual control literature may provide adequate guidance.

### Exercising the Model

As an example of the utility of the model of driver steering behavior, we can analyze the same vehicle used in the previous section, but now including a four-wheel steering system. One "control law" which has been proposed for such a steering system, e.g., [20], automatically determines the steering angle of the rear wheels so that the component of the vehicle velocity in the $y_2$ body axis remains zero (no vehicle "sideslip") when the driver turns the front wheels in normal maneuvering. Fig. 9 compares predicted driver/vehicle crossover frequencies $\omega_c$ at a number of velocities for the vehicle with conventional two-wheel steering and the same vehicle with the four-wheel steering system just described. In both cases, the control theoretic model for driver steering behavior was formulated with gain and phase margins for $\gamma_1/\gamma_2$, of at least 25°, and 6 dB, respectively. The superiority of the four-wheel system is evident in the results. As can be seen from the figure, crossover frequency, and hence, maneuverability, for the two-wheel system decreases with increasing velocity, while that for the four-wheel system, does not. Of course, care must be exercised in extrapolating the results of such an analysis to maneuvers of a severity that would invalidate the simplifying assumptions of the linear vehicle model used here.

### Conclusions

The control theoretic model of driver steering behavior has been developed consisting of low- and high-frequency compensation elements, with the latter obtained from application of a structural model of the human. A single parameter, the crossover frequency of the open-loop return ratio, was used to tune model response statistics to those from a simulation experiment involving lane keeping on a curving roadway. The remaining parameters in the driver model were selected on the basis of well-established feedback con-
control design principles dependent only upon the vehicle dynamics. A simple visual guidance cue can account for the manner in which the driver closes the outer feedback loop in steering tasks. Research is currently underway in employing the model to study the effects of different vehicle steering systems on handling qualities.

References


Ronald A. Hess received the B.S., M.S., and Ph.D. degrees in aerospace engineering from the University of Cincinnati, in 1965, 1967, and 1970, respectively. In 1982 he joined the faculty of the Department of Mechanical, Aeronautical, and Materials Engineering at the University of California, Davis, where is currently a Professor in the Division of Aeronautical Science and Engineering. His current research interests lie in the areas of automatic and manual control and in man/machine systems. He is an Associate Fellow of the AIAA, a member of IEEE, Sigma Xi, and Tau Beta Pi and is an Associate Editor of the Journal of Aircraft, and the IEEE Transactions on Systems, Man, and Cybernetics. He is a Vice-President of the IEEE Systems, Man, and Cybernetics and chairman of the Society's Manual Control Technical Committee. He is a member of the AIAA Technical Committee on Atmospheric Flight Mechanics.

Ali Modjahebedzadeh was born in 1960 in Iran. He received the B.S. degree in mechanical engineering from the University of California, Irvine, in 1983, the M.S. degree in mechanical engineering from San Jose State University in 1985, and is presently working towards the Ph.D. degree in the Department of Mechanical, Aeronautical, and Materials Engineering at the University of California, Davis. His research interests are focused in system dynamics and control, with emphasis upon mathematical modeling and control design. His current research area is manual control theory, applied to vehicular control.