Adaptive Control of a Single-Link Flexible Manipulator

Vincente Feliu, Kulidp S. Rattan, and H. Benjamin Brown, Jr.

ABSTRACT: A new method to control single-link lightweight flexible manipulators is proposed in this article. The objective is to control the tip position of the flexible manipulator in the presence of joint friction and changes in payload. Both linear and nonlinear frictions are overcome by using a very robust control scheme for flexible manipulators. The control scheme is based on two nested feedback loops: an inner loop, to control the position of the motor; and an outer loop, to control the tip position. Compensation for changes in load is achieved by decoupling the dynamics of the system and then applying a very simple adaptive control for the tip position. This results in a simple control law that needs minimal computing effort and, thus, can be used for real-time control of flexible arms.

Introduction

A major research effort has been made in the last 5 years to control flexible structures and, in particular, flexible arms. Several papers on the control of single-link flexible arms with a fixed payload at the tip have been published. These methods allow precise control of the tip position by sensing some states of the motor and the tip (position, velocity, etc.). All the states of the system are reconstructed from the measurements and are used to place the closed-loop poles of the arm. The reconstruction (by using filters or observers) usually involves a large amount of computation, especially when there is a high level of noise in the measurement.

Some adaptive control schemes have been proposed to handle the problem of a changing payload. These methods are based on Model Reference Adaptive Control (MRAC) or a two-stage process in which a system identification stage is followed by the adaptation of the controller as a function of the identified system parameters. Both methods require a large amount of calculations to be performed in real time and, hence, require a relatively powerful computer. These adaptive control methods are too complex to compensate for only one varying parameter. Moreover, nonlinear and time-varying joint friction, which plays an important role in many robots, is the parameter that has not been considered in these adaptive and nonadaptive control schemes.

This article presents a new control scheme that compensates for frictional effects and undesired changes in the dynamics of the system caused by changes in payload. This control method has been designed for the special case of lightweight (compared with the load that they handle) flexible arms. In this case, the mechanical structure exhibits a dominant low-frequency vibrational mode and negligible higher frequency modes.

Problems caused by Coulomb friction (which is a nonlinear component of the friction) as well as changes in the dynamic friction coefficient are overcome by using a general robust control scheme developed in [13]. This is composed of two nested loops: an inner loop that controls the position of the motor and an outer loop that controls the tip position (see Fig. 1). In this figure, \( \Phi(t) \) is the angle of the motor, \( \phi(t) \) is the tip position angle, and \( i \) is the motor current.

A new scheme to adapt the control law to changes in the load is proposed. The adaptive control scheme proposed here is based on the two-stage process described above. It uses a decoupling scheme (developed in [14]) that transforms the dynamic equations of the flexible arm into a simple double integrator system with only one parameter to estimate—the gain. This makes the identification stage very fast and the adaptation law very simple.

The control loop for the motor position is briefly described in the following section. Next, the adaptive scheme for the tip position loop is described. Experimental results are shown and followed by the conclusions.

Motor Position Control Loop

The motor position control loop corresponds to the inner loop of Fig. 1. We want to achieve two objectives in designing a controller for this loop:

1. To remove the modeling error and the nonlinearities introduced by Coulomb friction and changes in the coefficient of the dynamic friction.
2. To make the response of the motor position much faster than the response of the tip position control loop (outer loop in Fig. 1).

The fulfillment of the second objective allows us to substitute the inner loop by an equivalent block whose transfer function is approximately equal to one; i.e., the error in motor position is small and is quickly removed. This simplifies the design of the outer loop, as illustrated in the next section.

The differential equation relating the angle of the motor to the applied current can be written as in Eq. (1) where \( K \) is the electro-mechanical constant of the motor, \( i \) is the current of the motor, \( \phi(t) \) is the angle of

\[
\Phi(t) = \frac{1}{K} i(t)
\]

Fig. 1. General robust control scheme for flexible manipulators.
the motor, \( J \) is the polar moment of inertia of the motor and hub, \( V \) is the dynamic friction coefficient, \( C_i(t) \) is the coupling torque between the motor and the link (the bending moment at the base of the link), \( CF \) is the Coulomb friction, and \( t \) is time.

\[
Ki = J(d^2\theta_m(t)/dt^2) + V(d\theta_m(t)/dt) + C_i(t) + CF
\]

To simplify the design of the inner loop, the system described in Eq. (1) can be linearized by compensating for the Coulomb friction and decoupled from the dynamics of the beam by compensating for the coupling torque. This is done by adding, to the control current, the current equivalent to these torques (Fig. 2) and is given by

\[
i_c(t) = (C_i(t) + CF) \text{(sign of motor velocity)}/K
\]

The magnitude of the Coulomb friction, \( CF \), is identified from the spectral analysis of the motor position and the current signals. The details of the identification method are given in [15]. The coupling torque \( C_i(t) \) can be calculated either from strain gauge measurements at the link’s base or by the difference in angle measurements of the motor and tip. The second approach is used here. Because the beam is nearly massless, we can assume that \( C_i(t) = C(\theta_m(t) - \theta(t)) \), where \( C = (3 \cdot E \cdot I)/L \) is a constant that depends on the stiffness \( E \cdot I \) and length \( L \) of the arm. After compensating for the friction and coupling torque, the transfer function between the angle of the motor and the current is given by

\[
\frac{\theta_m(s)}{i(s)} = G_w(s) = \frac{K/J}{s(s + V/J)}
\]

The block diagram of the inner-loop control system is shown in Fig. 2 (discrete control version). The series and feedback controllers \( (G_1 \text{ and } G_2, \text{ respectively}) \) are designed so that the response of the inner loop (position control of the motor) is significantly faster than the response of the outer loop (position control of the tip) without any overshoot. This is done by making the gain of the series controller large and is limited only by the saturation current of the servo amplifier. It was shown in [13] that, in theory, this gain could be made arbitrarily large, even in the case of the arm being a nonminimum-phase system. It was also shown that large gain in this loop reduces the effects of nonlinearities caused by friction.

When the closed-loop gain of the inner loop is sufficiently high, the motor position will track the reference position with small error. The dynamics of the inner loop may then be approximated by "1" when designing the outer-loop controller.

**Tip Position Adaptive Controller**

Provided that the inner loop has been satisfactorily closed, the dynamics of the arm \( G_w(s) \) in Fig. 1 for the case of lightweight flexible arms are reduced to the following:

\[
G_w(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n} + \frac{\omega_d^2}{s}
\]

The natural resonant frequency of the beam with the motor fixed is \( \omega_n \) rad/sec and is related to the constant \( C \), the load at tip \( \omega_n \), and the length \( L \) by the following expression:

\[
\omega_n^2 = C/(mL^2)
\]

The proposed control scheme for the tip position is composed of three loops: a decoupling loop, a classical proportional plus derivative (PD) controller with a feedforward term (see Fig. 3), and an adaptation loop (shown in Fig. 4).

**Decoupling Loop**

The purpose of this loop is to simplify the dynamics of the arm. For the case of a beam with only one vibrational mode, a very simple decoupling loop can be implemented, which reduces the dynamics of the system to a double integrator. This is done by simply closing a positive feedback unity gain loop around the tip position. Thus, expression (4) is transformed into

\[
G'(s) = \frac{\omega_d^2}{s^2} + \frac{1}{s}
\]

**PD Controller**

It is well known that the poles of a plant of the form of Eq. (6) can be perfectly placed by using a simple PD controller [16]. Let us express this controller as

\[
G_{c3}(s) = \frac{\omega_d^2}{s}(K_P + sK_I)
\]

A feedforward acceleration term may be added in order to make the tip follow the reference without any delay and is given by

\[
\text{Feedforward}(s) = \frac{\omega_d^2}{s} \theta_d(s)
\]

where \( \theta_d(s) \) is the Laplace transform of the commanded trajectory profile for the tip.

Notice that (8) uses the second derivative of the reference. It means that parabolic profiles of at least, order 2 must be used as reference signals in this scheme. The resulting control scheme, after closing the decoupling and PD loops described earlier, is shown in Fig. 3. If the load at the tip was constant, this scheme would provide a nearly perfect trajectory tracking and error compensation for the tip position.

**Adaptive Control**

Changes in the carried load produce changes in \( \omega_m \), thus deteriorating the dynamic performance. An adaptive controller is proposed here to overcome this. It consists of two parts: the first identifies the gain of the system \( \omega_d^2 \) and the second changes the scaling block \( 1/\omega_d^2 \) of the controller (see Fig. 3) to the new estimated value. Estimation of the parameter \( \omega_d^2 \) is easily done by integrating the input, \( x(t) \), twice and then dividing the actual tip position by this value. Notice that

\[
\frac{\theta_m(t)}{x(t)} = \frac{\omega_0^2}{(x(t)/x(t)^2)}
\]

where \( \omega_0^2 \) refers to the estimate. A scheme to control the tip position of the arm, which includes the adaptive control law (third loop), is shown in Fig. 4. Notice that the gain of the plant can be estimated easily due to having previously decoupled the system. The tuning law for the controller (which consists of the scaling block of Fig. 4) is also very simple because of this reason.

**Experimental Results**

In this section, we first describe the experimental setup of a one-link lightweight flexible manipulator built in our laboratory. Next, the experimental results obtained from our control scheme are presented.
Experimental Setup

The mechanical system consists of a DC motor, a slender link attached to the motor hub, and a mass at the end of the link floating on an air table. Figure 5 shows the major parts of the system. The link is a piece of music wire (7 in. long and 0.032 in. in diameter) clamped in the motor hub. The tip mass is a \( \frac{1}{2} \)-in. thick, \( \frac{3}{4} \)-in.-diam. fiberglass disk attached at its center to the end of the link with a freely pivoted pin joint. The disk has a mass of 54 g and floats on the horizontal air table with minimal friction. Since the mass of the link is small compared to that of the disk, and because the pinned joint prevents generation of torque at the end of the link, the mechanical system behaves practically like an ideal, single-degree-of-freedom, undamped spring-mass system.

A direct-drive motor drives the link. The motor is powered by a 40-V power supply through a DC servo amplifier. The amplifier current limit is set to 4.12 A, which corresponds to a 9-lb. in. motor torque. Coulomb friction of the motor is about 0.288 Ib. in. (corresponding to 0.132 A) and has a significant effect on the control when the torque applied to the arm is low, as with our slender arm. The system was designed to give a tip response that is much slower than the motor response. Mechanical stops limit the travel of the motor and the hub to about 27 deg.

Two sensors are used for the control of the system. A 7\( \frac{1}{8} \)-in., 360-deg. potentiometer provides the angle of the motor shaft. A Hamamatsu tracking camera (with an infrared filter) senses the X-Y position of an infrared LED mounted on the tip of the arm. The workspace of the arm is limited to about \( \pm 3 \) in. (\( \pm 25 \) deg.) by the field of view of the camera.

The control algorithm is implemented on an MC68000-based computer with 512K bytes of dynamic RAM and a 10-MHz clock. Analog interfacing is provided with 12-bit A/D and D/A boards. Switch signals for starting and stopping control routines, as well as other functions, are read through parallel I/O ports. As floating-point operations are slow (approximately 0.5 msec per multiplication), real-time computations are done in integer (approximately 0.08 msec per multiplication) or short integer (0.02 msec) arithmetic. A matrix graphics interface card permits the display of data on a 12-in. monitor.

By using an identification technique described in [15], we determine the parameters of the arm (Fig. 6) to be

\[
\begin{align*}
J &= 0.005529 \text{ lb in. sec}^2 \\
V &= 0.01216 \text{ lb in./rad/sec} \\
K &= 2.184 \text{ lb in./A} \\
\text{Coulomb friction} &= 0.2883 \text{ lb in. (0.132 A)} \\
C_r(t) &= C(\theta_m(t) - \theta_i(t)) \\
C &= 0.674 \text{ lb in./rad}
\end{align*}
\]

The transfer function of the beam is given by

\[G_b(s) = \frac{43.75}{s^2 + 43.75}\]

The estimated value of the Coulomb friction corresponds to an equivalent torque generated by a beam deflection of 25 deg.; so its effect is very noticeable.

Inner-Loop Control Design

The inner loop incorporates compensation terms for Coulomb friction and the coupling between the motor and the beam, according to (2). The scheme of Fig. 2 is used for the inner loop. A delay term is included in the scheme to take into account the delay in the control signal because of the computations. A sampling period of 3 msec is used for this inner loop.

An optimization program was developed to get the best controllers using the model obtained for the motor. The settling time (considering an error of less than 1 percent) of the response of the motor to step commands in the motor angle reference input was minimized. The saturation limit of the current amplifier was also taken into account in this design. Step inputs were assumed as references for the inner loop because, in order to get a good control action, the command angle for the motor should experience very sharp changes. In fact, in our experiments,
the motor angle varied much faster than the angle of the tip. The resulting controllers were \( G_1(z) = 17.442 - 2.442z^{-1} \) and \( G_2(z) = 6.667 - 5.667z^{-1} \).

Figure 7 shows the response of the motor position to a step change in its reference, keeping the tip of the arm fixed in the zero angle position. This means that, in the steady state of this experiment, there is always a coupling torque \( C \) caused by the bending of 200 mrad existing between the angle of the motor and the angle of the tip. The zero steady-state error shown by the experimental data demonstrates the effectiveness of compensation achieved for the Coulomb friction and for the coupling of the motor with the beam. The settling time achieved for the motor is 33 msec, which is significantly faster than the dynamics of the beam. This allows us to assume that the equivalent transfer function of the inner loop is 1.

Outer-Loop Control Design

We first design the controller \( G_3(z) \) of Figs. 3 and 4 for the case of the load being 54 g. By designing an analog PD controller and then discretizing it, using the Tustin transform [17], we get the digital controller

\[
G_3(z) = 3281.25(1 - 0.987z^{-1}) + (1 - 0.74z^{-1}) (11)
\]

Adaptation

The adaptation scheme described previously in the adaptive control section is used. Parabolic profiles of order 2 are used as reference input for the tip position control system. Comparisons between the responses of the arm when using the nominal controller (9) (block \( 1/\omega_n^2 \) is tuned) were carried out. The response of the system with a nominal payload of 54 g and the nominal controller (nonadaptive) is very good because \( G_3 \) was designed for these conditions. The adaptive response with the nominal payload is essentially as good. Figure 8 shows both adaptive and nonadaptive responses when the payload is 142 g; Fig. 9 shows these responses when the payload is 15.73 g. Notice that the system without the adaptive controller becomes unstable in the last case.

**Conclusions**

A new method to control single-link lightweight flexible arms in the presence of joint friction and changes in the load is presented in this article. Effects of nonlinear friction are removed by closing a high gain loop around the motor position. This was developed in a previous paper [13] and includes compensating terms for the coupling torque and for Coulomb friction. The control of the tip position, when there are changes in the tip load, is carried out by first decoupling the dynamics of the system by closing a unity positive feedback loop around the tip position and then designing an adaptive controller for the decoupled system. The decoupling makes the adaptive control very simple. The general control scheme is shown to be simple and computationally efficient. The controller is composed of three nested control loops plus an adaptation loop, but each consists of very simple elements. In fact, our experiments show that when using a computer of very modest capabilities, a controller that fulfills the desired specifications can be implemented. The experimental responses were shown to be good even in the case of extreme conditions; the Coulomb friction was very high and changes in the payload were about three times more and less than the nominal load of 54 g.

Another advantage—from the design point of view—is that each loop is designed independently (starting from the inner loop), and their elements are calculated easily and according to simple specifications. The inner loop is designed to compensate for friction and to make the motor response fast. Both goals are achieved with the same high gain PD controller. The middle loop decouples the dynamics of the system (reduces its transfer function to a double integrator). The outer loop gives a fast and accurate response of the tip position (a simple PD with a feedforward term). The adaptive controller takes care of changes in the load by estimating only one parameter of the system.

Finally, this control approach is different from others in that the existing methods to control flexible arms are based on explicit control of the tip position, where the controller generates the current for the DC motor of the joint as a control signal. The proposed method is based on the simultaneous explicit control of the joint motor position and tip position. The controller for the motor position generates a control signal that is a current for the DC motor, as in the other existing methods, while the tip position controller generates a control signal, which is a motor position reference for the inner loop.
References


Vincent Feliu received his degree in industrial engineering from the Polytechnical University of Madrid in 1978 (Honors) and his Ph.D. from the same university (Premio Extraordinario) in 1982. He joined the Electrical Engineering Department of the Universidad Nacional de Educacion a Distancia in 1980. Since then, he has worked as an Associate Professor. He has taught a variety of courses in real-time control and robotics. He has been a Fulbright Scholar in the last 2 years in the Robotics Institute of Carnegie-Mellon University. His research interests include multi-variable and digital control systems, kinematic and dynamic control of robots, and modeling and identification techniques.

Kuldip S. Rattan received the B.S. degree in electrical engineering (with honors) from Punjab Engineering College, Chandigarh, India, in 1969 and the M.S. and Ph.D. degrees in electrical engineering in 1972 and 1975, respectively, from the University of Kentucky, Lexington. From 1976 to 1978, he was a Research Associate in the Department of Physiology and Biophysics, University of Kentucky. He joined the Wright State University as an Assistant Professor in 1979 and currently holds the rank of a Professor of Electrical and Computer Engineering. During the 1987-1988 academic year, he was a Visiting Professor at Carnegie-Mellon University's Robotics Institute. His current research interests include robotics, digital control, CAD/CAM, and microprocessor applications. He has published over 70 technical papers in related areas. Dr. Rattan is Past President of the IEEE Control Systems Society (Dayton Chapter) and is a Senior Member of the IEEE.