Nonlinear Identification of Hydraulic Servo-Drive Systems

Mohieddine Jelali and Helmut Schwarz

This article deals with the identification of nonlinear models in observer canonical form of hydraulic servo-drives from sampled data of input-output measurements. The data are processed by a modified Recursive Instrumental Variables algorithm, to provide input-output relationships of the plant dynamics. From the parameters of the input-output relations, continuous state-space nonlinear models can be derived. Tests were performed on a strongly nonlinear hydraulic drive which has been used for research in our laboratory for several years. Results demonstrate good correspondence between the data and the identified models.

Introduction

Hydraulic servo-drives are used in many industrial plants, because they can produce large forces and torques with high speeds. However, the rather complex structure of such drive systems makes it difficult to develop suitable, preferably low-order models of the dynamic of the plant. The models are needed for the design of state observers, filters, and controllers. The design is most simplified if the model of the plant has a nonlinear canonical form [1,2]. In actual hardware, however, systems rarely have these suitable forms. Nonlinear transformations into canonical forms therefore must first be determined under rigorous conditions and with considerable mathematical effort (integration of partial differential equations and inversion of nonlinear algebraic equations). To avoid this, the practical application of system identification techniques provides satisfactory models of individual units in some desired form.

The aim of the research presented in this article is to obtain models of a hydraulic servo-drive directly, in the nonlinear observer canonical form, via parameter identification. In recent years, much effort has been devoted to modeling of hydraulic systems using bilinear models. Several of these models have been evaluated by tests on real plants, and are well established [3,4,5]. However, the identification methods used, the maximum likelihood method and prediction error method, require suitably specified ("good enough") initial values of the unknown parameters and states of the system. An unsuitable choice causes convergence and singularity problems that, in real applications, are very difficult to solve.

In this article, the parameter estimation is based on a modified Recursive Instrumental Variables algorithm that enables us to overcome the difficulties mentioned above. We consider state quadratic nonlinearities for better modeling of the real dynamics of hydraulic drives. For handling time derivatives of measurements, the so-called Linear Integral Filter proposed by Sagara and Zhao [6] is used. The identification procedure is applied to an experimental setup. A good correspondence is obtained between the data and the models which are identified directly in nonlinear, especially quadratic, observer canonical form.

Description of the Hydraulic Drive

The physical process used as testing bench consists of a servo valve and a hydraulic cylinder coupled with a moving mass. Fig. 1 illustrates the test stand used in this study. In order to avoid the representation of many equations which may be found, for instance, in Dietz and Prochnio [7] and Koeckemann [8], a schematic diagram of the system is shown in Fig. 2, and a detailed block diagram is given in Fig. 3. The input signal of the system is the voltage \( u \) and the output signal is the position \( x \) of the moving mass \( m \). The state variables are listed in Table 1.

The most significant nonlinearities of the plant are the multipliers, the square root functions, the oil elasticity and the friction. In practice, it is difficult to determine the physical parameters associated with these nonlinearities. Thus, system identification techniques are needed to obtain approximate models of the system such that the error between measured data and model is minimized.

<table>
<thead>
<tr>
<th>Table 1. Symbol and State Definitions</th>
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<tr>
<td>Symbols</td>
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<tr>
<td>( V_1, V_2 )</td>
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<td>( P_1, P_2 )</td>
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<tr>
<td>( Q_{1}, Q_{2} )</td>
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<td>( Q_{L} )</td>
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<td>States</td>
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The multiple integral of a continuous-time signal $z(t)$ is defined by

$$I_n z(t) = \int_{n-l-T}^{n-1-T} \cdots \int_{l-i-T}^{l-i-1-T} z(t_n) dt_n dt_{n-1} \cdots dt_1$$

where $T$ is the sampling interval and $l$ is the length factor of the LIF.

The multiple integral of the derivative $z''(t) = d^2 z(t)/dt^2$ of the signal $z(t)$ can be approximately calculated by

$$I_n z''(t) = \sum_{i=0}^{nl} p_i(t) = \sum_{i=0}^{nl} p_i(T)$$

where $q^{-i} z(t) : = z(t - iT)$ is the delay operator. In discrete form (sampled $z_k$) this becomes

$$I_n z_k^{(j)} = \sum_{i=0}^{nl} p_i z_k$$

where

$$f_j = (1 - q^{-1})^j (f_0 + f_1 q^{-1} + \cdots + f_n q^{-n-j})$$

and $f_i$ are the coefficients determined by a formula of numerical integration, e.g., the trapezoidal rule

$$f_0 = f_1 = T, f_i = T; i = 1, 2, ..., l - 1.$$  \hfill (7)

The $p_i$ ($i = 0, 1, ..., nl; j = 0, 1, ..., n$) in (3) and (5) are the coefficients of the polynomials $f_j$ in (6).

If the input signal is constant in each interval $[t_k - iT, t_k]$, then

$$I_n \left[ z_k(t)^{(j)} \right] = \sum_{i=0}^{nl} p_i(t)$$

Nonlinear Observer Canonical Form

The nonlinear observer canonical form (NOCF) is defined by Keller [9] as follows:

$$\ddot{\mathbf{x}}(t) = \begin{bmatrix} -a_1(\mathbf{x}) \\ \vdots \\ -a_n(\mathbf{x}) \end{bmatrix} + \begin{bmatrix} b_1(\mathbf{x}) \\ \vdots \\ b_n(\mathbf{x}) \end{bmatrix} u(t) = E_n \ddot{\mathbf{x}} - a(\mathbf{x}) + b(\mathbf{x}) u(t)$$

$$y(t) = \mathbf{x}(t) : \ddot{\mathbf{x}}(t_0) = \tilde{\mathbf{x}}_0$$

Identification Method

The continuous parameter estimation from sampled data of input-output measurements involves the problem of computing derivatives of measurements. For this, Sagara and Zhao [6] proposed an operation of numerical integration, the so-called Linear Integral Filter (LIF), for linear differential equations. This method will be extended with the goal to identify some linear-in-parameters nonlinear systems like those in observer canonical form.

Linear Integral Filter

Some characteristic properties of the LIF (for more details see [6]) needed here are briefly given in the following:

- The multiple integral of a continuous-time signal $z(t)$ is defined by

- The multiple integral of the derivative $z''(t) = d^2 z(t)/dt^2$ of the signal $z(t)$ can be approximately calculated by

- If the input signal is constant in each interval $[t_k - iT, t_k]$, then

- Nonlinear Observer Canonical Form

The nonlinear observer canonical form (NOCF) is defined by Keller [9] as follows:
The observer canonical form derives its name from the property that a nonlinear observer can be constructed in canonical coordinates as in the linear case by an eigenvalue assignment. For the NOCF an input-output relationship has to be found such that the LIF can be applied. Since the nonlinear functions $a_i(x_n)$ and $b_i(x_n)$ in $(9)$ can be approximated by the Taylor series expansion

$$a_i(x_n) = \sum_{j=0}^{l_i} \alpha_{ij} x_n^j$$
$$b_i(x_n) = \sum_{j=0}^{l_i} \beta_{ij} x_n^j$$

it can be shown from the state space description $(9)$ and using $(10)$ that

$$\dot{x}_n^{(n)}(t) = -\sum_{i=1}^{n} \sum_{j=1}^{l_i} \alpha_{ij} x_n^{(i-1)} + \sum_{j=0}^{l_i} \beta_{ij} x_n^j u_k^{(i-1)} + e_k$$

The multiple integration of $(11)$, taking the sampled data $u_k, y_k$ and using the LIF, results in the following expression

$$\mathcal{F}_n y_k = -\sum_{i=1}^{n} \sum_{j=1}^{l_i} \alpha_{ij} \mathcal{F}_{n-1} x_n^{(i-1)} + \sum_{j=0}^{l_i} \beta_{ij} \mathcal{F}_{n-1} (y_k u_k)^j + e_k$$

(12)

where $e_k$ is the equation error, which is composed of the truncation error due to numerical integration and the noise term due to the noisy output signals.

Commonly, only the linear terms ($l_{ai} = l_{bi} = 1$) in $(10)$ are considered. The higher-order terms are thus ignored following the assumption that they are negligible when the systems state close to the reference point chosen for the linearization. In this article we go two steps further by taking into account also the bilinear and the quadratic terms. Higher-order terms could also be considered but, since they bring little improvement to the quadratic approximation while adding a lot to the computational burden, they will be left aside in the application on the hydraulic drive presented here. Nevertheless, the identification method will be derived for any $l_{ai}, l_{bi}$.

Furthermore, the filter parameter $\lambda$ affects considerably the accuracy of the parameter estimation. It is pointed out by Sugara and Zhao [6] that $\lambda$ should be chosen so that the frequency bandwidth of the LIF matches as closely as possible the frequency band of the system. In practical use, however, a-priori information about the frequency band of the system are often not available. Therefore, many identification experiment trials must be taken.

Recursive Instrumental Variables Algorithm

Define the true parameter vector

$$\theta^T = [\alpha_{11}, \alpha_{21}, \ldots, \alpha_{n1}; \alpha_{12}, \alpha_{22}, \ldots, \alpha_{n2}; \ldots; \alpha_{1l_i}, \alpha_{2l_i}, \ldots, \alpha_{nl_i}; \beta_{10}, \beta_{20}, \ldots, \beta_{n0}; \beta_{11}, \beta_{21}, \ldots, \beta_{1l_i}, \beta_{2l_i}, \ldots, \beta_{nl_i}]$$

(13)

and the data vector

$$\mathcal{F}_n y_k = \{\mathcal{F}_n y_k, \ldots, \mathcal{F}_{n-1} y_k, \mathcal{F}_{n-2} y_k, \ldots, \mathcal{F}_0 y_k, \mathcal{F}_n u_k, \ldots, \mathcal{F}_{n-1} u_k, \mathcal{F}_{n-2} u_k, \ldots, \mathcal{F}_0 u_k\}$$

Thus, $(12)$ can be rewritten as

$$\mathcal{F}_n y_k = \varphi_k^T \theta + e_k$$

(15)

and least-squares identification methods can be used.

One very effective method is to use the recursive instrumental variable (IV) method, which is asymptotically unbiased for a suitable choice of the IV and does not require a priori knowledge of the noise statistics. The following algorithm is given by Ljung and Soederstrom [10]:

$$\hat{\theta}_k = \hat{\theta}_{k-1} + L_k \left( \mathcal{F}_n y_k - \varphi_k^T \hat{\theta}_{k-1} \right)$$

and

$$\gamma_k = \lambda \gamma_{k-1} + (1 - \lambda) \lambda_k$$

$$\lambda_k = \lambda_{k-1} + \left( 1 - \lambda_{k-1} \right)$$

$$\varphi_k = \varphi_{k-1} - \lambda_{k-1} \gamma_k / \lambda_k$$

$$L_k = \frac{P_{k-1} \varphi_k}{\gamma_k + \varphi_k^T P_{k-1} \varphi_k}$$

$$\hat{\theta}_0 = 0$$

$$\lambda_0 = 0.99$$

$$\lambda_0 = 0.95$$

(16)

where $\lambda, P, N$ are the forgetting factor, the covariance matrix, and the number of data pairs, respectively. $\varphi$ is a large number and $I_n$ is the identity matrix. The IV vector $\varphi_k$ must be highly correlated with the system signals, but not with the noise which leads to biased parameter estimates. The IV are formed in different ways from (filtered) inputs, delayed and/or outputs as well as (filtered) set point variations. Several IV variants are described and compared by Soederstrom and Stoica [11]. One choice of the IV, consisting of a combination of delayed inputs, is for example

$$\xi_k = \varphi_k |_{x_n = \xi_{n-k}}$$

(17)

where $k_0$ is a delay parameter. The cost function to be minimized by the algorithm is, for $N$ observations,

$$J(\theta) = \sum_{k=1}^{N} \| \mathcal{F}_n y_k - \varphi_k^T \theta \|^2$$

(18)

The IV estimates are locally convergent to the true parameters, in general, and the convergence of the algorithm $(16)$ is considerably fast. For a detailed treatment of convergence and stability of recursive IV algorithms, the reader would consult Ljung and Soederstrom [10].

Experimental Setup and Results

The experimental setup considered here is composed of a hydraulic cylinder, a servo valve, and a digital computer. The
Fig. 4. Data acquisition system.

The block diagram in Fig. 4 illustrates the data acquisition system. The cylinder moves the mass \( m = 5 \text{ kg} \) depending on the oil flows \( Q_1 \) and \( Q_2 \) (in the chambers \( A \) and \( B \)) which are managed by the valve. The voltage \( u \) of the servo valve is obtained via a RTI-815-interface card (Analog Devices) through a measuring amplifier. The RTI-815 works as a 12-bit digital-to-analog (D/A) converter (in a 386-PC), which is scaled to command \( \pm 10 \text{ V} \). An incremental position measuring system (IK-120 card, Heidenhain) provides the position measurement \( x \) to the computer.

Due to the fact that the hydraulic system has an integrating behavior with regard to the position of the cylinder, and since the identification has to be stable at each step \( k \), the velocity is used as the output signal \( y \) for the identification. Thus, the measured position is numerically differentiated using the difference equation

\[
y_k = \frac{x_k - x_{k-1}}{T}.
\]

This reconstruction enhances high-frequency noise (see, for example Figs. 6, 7, and 8). The sampling rate was \( T = 1 \text{ ms} \). Of course, the high-frequency noise can be removed by smoothing or filtering. This is not necessary here since the LIF works as a pre-filter and overcomes noisy signals.

The input signal is normalized in the region \([-1,+1]\]. In order to obtain the most information possible about the relevant plant dynamics, the input test signal has to be designed in such a way that it varies over the entire admissible region. A random amplitude input with constant period (see Fig. 5) was applied to the real plant.

In order to show the influence of the algorithm parameters (the length factor \( l \) of the LIF and the delay parameter \( k_0 \)) on the models quality we varied them stepwise. Figs. 6, 7, and 8 show the identified and measured responses of the system for three resulting models in quadratic observer canonical form \((n = 4)\), that is,

\[
\ddot{x}(t) = E_n\dot{x}(t) - \alpha_1\ddot{x}(t) - \alpha_2\dddot{x}(t) + \beta_0u(t) + \beta_1\ddot{x}(t)u(t)
\]

\[
y(t) = \dddot{x}(t).
\]
Table 2. Estimated Parameters of Some Identified Models

<table>
<thead>
<tr>
<th></th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
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<tr>
<td>( l )</td>
<td>22</td>
<td>23</td>
<td>25</td>
</tr>
<tr>
<td>( k_0 )</td>
<td>7</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

\( \alpha_1 [10^{-3}] \)

\[
\begin{bmatrix}
0.1 \\
7.3 \\
51.8 \\
266.7
\end{bmatrix}
\begin{bmatrix}
0.1 \\
5.9 \\
45.5 \\
244.0
\end{bmatrix}
\begin{bmatrix}
0.03 \\
4.4 \\
38.5 \\
204.1
\end{bmatrix}
\]

\( \alpha_2 [10^{-3}] \)

\[
\begin{bmatrix}
0.01 \\
0.4 \\
-2.1 \\
12.1
\end{bmatrix}
\begin{bmatrix}
0.01 \\
0.5 \\
0.49 \\
50.6
\end{bmatrix}
\begin{bmatrix}
0.0 \\
0.2 \\
-1.1 \\
7.3
\end{bmatrix}
\]

\( \beta_0 [10^{-3}] \)

\[
\begin{bmatrix}
0.1 \\
5.9 \\
48.9 \\
638.8
\end{bmatrix}
\begin{bmatrix}
0.1 \\
4.8 \\
7.0 \\
44.0
\end{bmatrix}
\begin{bmatrix}
0.03 \\
3.9 \\
8.4 \\
48.9
\end{bmatrix}
\]

\( \beta_1 [10^{-3}] \)

\[
\begin{bmatrix}
-0.04 \\
-2.9 \\
-70.5 \\
-125.1
\end{bmatrix}
\begin{bmatrix}
-0.03 \\
-2.3 \\
-58.0 \\
-91.2
\end{bmatrix}
\begin{bmatrix}
-0.01 \\
-1.7 \\
-47.9 \\
-147.1
\end{bmatrix}
\]

\( M N E \) 0.07 0.12 0.08

\( \| u(t) \| \) takes into account the fact that the dynamic behavior of \( y_k \) is approximately symmetric about zero. The estimated parameters (round values) of the models are given in Table 2.

The good correspondence between the measured data and the quadratic approximation demonstrates the efficiency of the presented identification method. In order to compare different models (for different \( l \) and \( k_0 \)) the mean normalized error

\[
M N E = \frac{\| y - \hat{y} \|}{\| y \|^2} \cdot 100\%
\]  

is considered where \( y \) is the measured and \( \hat{y} \) the estimated data vector. The results are summarized as follows:

- The choice of the design parameters \( l \) and \( k_0 \) have a great influence on the quality of the identified models. For the hydraulic drive presented here, \( l \) should be between 20 and 25 and the delay parameter \( k_0 \) between 5 and 10. However, not every combination involves a stable model. This can be shown by simulations.
- The quality of the identified models is worse for \( n < 4 \), but not significantly better for \( n > 4 \).
- The bilinear and quadratic dynamic of the plant must be considered for better modeling of the real dynamics of hydraulic drives.
- For a large number of observations (\( N > 1500 \)) the multiple integration of the input and output signals may be unstable so that the resulting models are not satisfactory. This is due to the fact that the LIF can be regarded as an unstable IIR (Infinite Impulse Response) Filter that has multiple poles on the unit circle. Therefore, the output of the LIF as well as the equation error will increase with the time. This problem may be solved by resetting the algorithm after a suitable period of time [12].

With the intention of assessing the true performance of the identification method, a common procedure that can be regarded as a test of the model's validity was applied. That is, the system is simulated with input signals other than those used for identification and compare measured output with the simulated model output. Exemplary comparisons between the measured output and the model output for model 3 are given in Figs. 10 and 11. Fig. 9 shows the input signals used. These and other validation tests have confirmed the good performance of the system identification method used in this study.

The errors in the responses of the simulated/identified model output, compared with measured output, are caused by some unmodeled effects like static friction, the deviation between the hydraulic and the electric zero point of the drive, as well as the decreasing of the supply pressure (which is neglected here). Nevertheless, the identified quadratic models enable us to avoid the complex physical model structure, with many unknown parameters, and it can be proven that they bring great improvement to linear approximations. Furthermore, the identification of the model directly in observer canonical form has the advantage that the state observers for the hydraulic system can be designed effortlessly by pole placement.

![Fig. 8. Plant and model 3 responses.](image)

![Fig. 9. Input signals used for model validation.](image)
An experimental identification method for linear-in-parameters continuous-time nonlinear systems, like those in observer canonical form, was presented. The method is based on a modified Recursive Instrumental Variables algorithm and the Linear Integral Filter for handling time derivatives of measurements.

The hydraulic servo-drive considered as a testing bench has strongly nonlinear dynamics. It has been shown that the quadratic modeling (observer canonical form) of this system is satisfactory. This has been confirmed by comparisons of plant and model responses.

The identification method can also be applied to get models of the hydraulic drive in other nonlinear canonical forms like the observability canonical form. This will be treated in future research.

Conclusion

An experimental identification method for linear-in-parameters continuous-time nonlinear systems, like those in observer canonical form, was presented. The method is based on a modified Recursive Instrumental Variables algorithm and the Linear Integral Filter for handling time derivatives of measurements.

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References


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