Stability of Dynamical Systems—Continuous, Discontinuous, and Discrete Systems

by ANTHONY N. MICHEL, LING HOU, and DERONG LIU

Reviewed by Alessandro Astolfi

have always believed that stability is the most important concept in systems and control theory. From the early days of my undergraduate studies, when I was first exposed to Lyapunov theory, to these days in which I devote a significant proportion of time to proving stability properties of dynamical systems and teaching stability to second- and third-year undergraduate students, I have never ceased to be fascinated by the elegance of this notion, its far-reaching implications, and its significance in applications.

It is fair to claim that control engineers and mathematicians concur in asserting that stability is a property that, in some form, should be possessed by every system, if the system is to have any practical relevance. For this reason numerous research papers, chapters of books, and entire books are devoted to the study of stability, its characterization, descriptions of its use in analysis and design, and extensions or variations of the standard notions of Lyapunov and Lagrange.


The reader wanting to learn stability from the classical books from the 1950s and the 1960s has to face a few serious obstacles. First, some of these books are in Russian (although an English translation is often available, see for example [7]); second, the notation, the terminology, the style, and even the logic are unfamiliar, and it may be extraordinarily hard, if not impossible, to extract the desired information, essentially since these older books often do not have an index. Finally, the class of systems and the problems that are of interest today are different from those studied in the past, which means that the result or the argument that we are interested in may not be available, or may be given in a form that is of little use.

I do not mean to detract from the classical body of literature on stability—I have learned stability using Hahn’s Stability of Motion [3], LaSalle and Lefschetz’s Stability by Lyapunov’s Direct Method [5], Carr’s Applications of Center Manifold Theory [2], and LaSalle’s The Stability of Dynamical Systems [7]—but rather I believe that textbooks on stability (similarly to everything else) require modernization, that is, the subject must be illustrated and taught in modern notation and set in the perspective of current problems.

This goal is partly achieved by existing books, for example, [1], [8], and [4]. Nevertheless, as stated in the preface of the book being reviewed, “there are no books on stability theory that are suitable to serve as a single source for the analysis of system models” that include, simultaneously, continuous-time and discrete-time components, finite-dimensional and distributed-parameter subsystems, and components described by continuous and discontinuous differential equations, that is, systems that are of current interest.

The goal of this book is to provide a reference text for graduate students and researchers on stability theory for the class of systems encountered in modern applications. In this respect, the goal is indeed achieved since the book offers a self-contained presentation of stability theory.

CONTENTS OF THE BOOK

The book is organized in nine chapters, most of which include sections with notes and references, problems, and bibliography. I like the structure of the book, although I believe that a single bibliography at the end of the book is preferable to having bibliographies at the end of each chapter. In addition, most of the problems and examples are somewhat too theoretical, such as problems and examples that ask the reader to out the proof of an unproved result or condition. This structure renders some parts of
the book very hard to digest for nonexperienced readers and reduces its pedagogical value.

The nine chapters can be clustered into six logical parts. The introduction, Chapter 1, provides a brief description of the notion of dynamical system and gives a concise view of the history of stability theory. In addition, this chapter specifies the goal of the book and its organization.

Chapter 2 is devoted to the formal definition of a dynamical system. Several classes of systems are presented, namely, systems described by ordinary differential or difference equations or inequalities, differential inclusions, integrodifferential equations, partial differential equations, and discontinuous differential equations. Special care is taken to highlight the key concepts and ingredients, and an effort is made to consider all classes of systems that are relevant to applications. In addition, several important issues illustrating properties of solutions are highlighted by means of elementary, yet carefully selected, examples.

Chapters 3 and 4 present the fundamental theoretical results, including direct and converse Lyapunov and Lagrange stability results, invariance theory, and comparison methods for the classes of systems introduced in Chapter 2. These two chapters are the core of the book, since the subsequent chapters primarily discuss applications of the general theory to specific problems or classes of systems. While reading these chapters the reader has the impression that the concepts and statements are somewhat repetitive. This impression is not correct since concepts and statements refer to different types of systems, and therefore there are significant differences. The authors have done an excellent job maintaining the rigor of the presentation, and in providing stand alone statements for diverse types of systems. On the other hand, I think they should have helped the reader more in identifying key differences between the various statements.

Chapter 5 deals with the application of the general theory to a class of discrete-event systems. Although very interesting, this short chapter breaks the logical flow of the book.

Chapters 6, 7, and 8 apply the general theory developed in chapters 3 and 4 to finite-dimensional systems, linear systems, and some classical problems, including the absolute stability problem as well as the stability of Hopfield neural networks, a class of digital control systems, pulse-width-modulated feedback systems, and a class of digital filters. The reader with some background on systems theory will find the content of these chapters familiar. Nevertheless, these chapters contain several nonstandard results and examples, thus making interesting reading even for experts.

Chapter 9 is devoted to the study of stability issues for infinite-dimensional (continuous and discontinuous) systems. This chapter is the most technical. It is notable that the authors include several well-selected and worked-out examples to help the reader grasp the key concepts and tools. The theoretical results are illustrated by means of several applications, including a kinetic model of a nuclear reactor and neural networks with delays.

CONCLUSIONS

The use of this book as a reference text in stability theory is facilitated by an extensive index. The table of contents would have been enhanced by the inclusion of third level headings.

In conclusion, Stability of Dynamical Systems—Continuous, Discontinuous, and Discrete Systems is a very interesting book, which complements the existing literature. The book is clearly written, and difficult concepts are illustrated by means of good examples. The book is suitable for readers with a solid mathematical background as well as some basic systems and control knowledge. The book should provide a useful reference for researchers working in control theory as well as for Ph.D. students.

REFERENCES


REVIEWER INFORMATION

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Linear Feedback Control—Analysis and Design with MATLAB

by DINGYÜ XUE, YANGQUAN CHEN, and DEREK P. ATHERTON

Reviewed by Guoxiang Gu

Because of the complexity of modern physical processes and the mathematical nature of many design methods, it can be difficult, if not impossible, to carry out control system design without the aid of computer analysis and simulation tools. Matlab has emerged as the leader among such tools. More than ten Matlab toolboxes have been developed to aid modeling, simulation, analysis, and synthesis in the design of feedback control systems. These toolboxes greatly facilitate the process of control system design.

Almost all of the textbooks in introductory control courses include Matlab examples and exercises. However, not many have integrated Matlab and the control content well. What is missing in many control textbooks is a natural and smooth flow of illustrative Matlab scripts inserted into the appropriate places in the presentation of the material. There is thus a need for a book that contains not only control theory but also adequate Matlab examples.

I believe that Linear Feedback Control—Analysis and Design with MATLAB, which has been written primarily as a reference book, fills this gap. By reducing the mathematics, increasing the number of Matlab-worked examples, and inserting short scripts and plots within the text, the authors have created a book for a wide range of users, including beginners in the field, students who wish to bridge the gap between control theory and the use of Matlab in the analysis and design of control systems, and practicing engineers.

CONTENTS

The book is organized into eight chapters, which cover modeling, analysis, simulation, and controller design. Each chapter contains Matlab examples with short scripts and plots inserted within the text. In addition, each chapter provides a large number of exercises that require the use of various Matlab commands or Matlab programming.

Chapter 1 introduces the concept and structure of feedback control systems and offers a brief but interesting historical account of control systems. In addition, this chapter describes the organization of the book and provides a short tutorial on the basic elements of Matlab.

Mathematical models for feedback control systems are developed in Chapter 2. The chapter begins with an electric circuit before introducing the Laplace transform. Transfer function models with Matlab representations are presented subsequently, covering both continuous-time and discrete-time systems, including multi-input/multi-output (MIMO) systems. State-space models with Matlab illustrations are also covered together with realization theory. Interconnections within a linear system as well as conversion between different mathematical models and between Matlab representations are described in detail. The chapter ends with a brief introduction to system identification, which is more involved mathematically.

Chapter 3 focuses on the analysis of linear control systems. Various system properties, such as stability, controllability, observability, and Kalman decomposition, are studied and analyzed. This material is followed by time-domain analysis that includes norm measures of signals and systems as well as responses of state-space systems. Many Matlab scripts are used to illustrate the use of relevant Matlab commands. Numerical simulation of linear systems is presented with emphasis on the step response of a second-order prototype system. Both qualitative specifications of the step response and the evaluation of the step response with Matlab are discussed. Root locus is introduced not only as a stability analysis tool but also as a controller design procedure, both illustrated with Matlab examples. An equally important design tool is the frequency-domain analysis that introduces the notion of gain and phase margins crucial to both performance analysis and controller design. The last topic of the chapter is model reduction, which echoes the identification section of the previous chapter.

Simulation analysis of nonlinear systems is covered in Chapter 4. This chapter is the shortest one but involves the most sophisticated aspects of mathematics and the Matlab toolbox. Simulink becomes the main tool for modeling and simulating nonlinear systems. The authors introduce Simulink first and then provide guidelines for Simulink modeling. Nonlinear modeling is illustrated by examples, using elements such as relay loop and double-valued nonlinearities. These examples help the reader to understand both the mathematical aspects of nonlinear systems and the programming side of Simulink. Linearization is treated at the end of the chapter.

The remaining four chapters are concerned with controller design. Chapter 5 focuses on model-based controller design. Strictly speaking the other chapters on controller design are also model based. The difference lies in that the design techniques in Chapter 5 are solely based on differential or difference equation models. The material in this chapter is rich and includes lead-lag synthesis based on the transfer function method and Bode plots with phase margin as the performance measure, for which SISOTool
from Matlab is a convenient and effective design tool; linear-quadratic optimal control based on the state-space method for synthesizing observer-based controllers to minimize a linear-quadratic performance index; pole placement design, which combines state feedback and observation to form an observer-based controller and covers a variety of effective algorithms; and decoupling of multivariable systems, which is a less significant part of the chapter.

Chapter 6 covers traditional proportional-integral-differential (PID) control. Several PID design techniques are presented. The chapter begins with discussions on the action of proportional, differential, and integral controls, followed by the Ziegler-Nichols tuning formula and other PID controller tuning algorithms. Again, Matlab tools are used to work out many examples in this chapter. This approach helps the reader to understand the basics of PID control and its tuning formulas. Applications to other types of plants are also covered, including tuning formulas for various types of plant models. The chapter includes anti-windup PID controllers and automatic tuning of PID controllers. In addition, solutions to constrained and unconstrained optimization problems are presented with application to optimal controller design in which a Matlab interface is used.

Chapter 7 focuses on optimal robust control system design. This chapter starts with linear-quadratic-Gaussian control and loop transfer recovery in addressing the stability margin problem, which are followed by the small gain theorem and uncertainty descriptions in the frequency domain. These materials are solid preparation for introducing $H_\infty$-based robust control. However, the presentation of $H_\infty$ is too brief to include the complete design formulæ for the general case, even though frequency weighting functions are discussed with weighted sensitivity minimization as an illustrative example. This problem is less severe in the case of $H_2$ control, which is also covered in the chapter.

The final chapter on controller design, Chapter 8, focuses on fractional-order controllers. Because of Matlab, analysis and synthesis of these feedback controllers is feasible. The chapter covers computations, analysis, modeling, reduction, and controller design.

CONCLUSIONS

This book is an excellent addition to the existing literature on control. The book is easy to read, although mathematical details are mostly omitted. Many Matlab examples are used to illustrate the material covered. All example scripts within the book, as well as the CtrlLAB package developed by the authors, are freely downloadable from Matlab Central. I recommend that every control instructor obtain a copy of the package in which many Matlab examples can be taught to complement the textbook and to meet students’ needs in learning Matlab. I also encourage practicing control engineers to obtain a copy of the book, which will provide them with a valuable source of information and a handy reference in control.

REVIEWER INFORMATION

Guoxiang Gu received his Ph.D. degree in electrical engineering from the University of Minnesota in 1988. From 1988 to 1990, he was with the Department of Electrical Engineering, Wright State University, Dayton, Ohio, as a visiting assistant professor. Since 1990, he has taught at Louisiana State University, where he is currently a professor. His research interests include control, system identification, and digital signal processing. He is an associate editor for Automatica and SIAM Journal on Control and Optimization.

Mathematical modeling of physiological systems is an interdisciplinary field that applies fundamental laws in mathematics, physics, chemistry, and engineering to characterize the interactions among physiological subsystems. These models are useful tools in various applications in medicine by supporting experimental design and facilitating better understanding of the functions underlying physiology [1], [2]. In addition, physiological models provide a convenient and cost-effective tool for medical training and education [3], and they assist clinical diagnosis, treatment, and medical device development [4], [5].

For example, the cardiovascular system consists of the heart, blood vessels, blood, and nerves. The heart pumps the blood into the blood vessels to circulate throughout the body and perfuse the organs. The nervous system controls the heart’s pumping action (heart rate and cardiac
contractility), the vascular resistance, and the blood volume to regulate arterial pressure, and thus maintain optimal organ perfusion. As described in *Cardiovascular and Respiratory Systems Modeling, Analysis, and Control*, cardiovascular hemodynamics can be modeled based on fluid mechanics, while blood pressure regulation due to nerve actions (known as the baroreflex) can be formulated as a feedback mechanism.

Efforts to develop physiological models have been ongoing for many years. Wiener examined the instability and oscillations of neurological control systems from the Volterra series perspective in the 1940s [6]. Guyton, Grodins, and coworkers studied the cardiovascular and respiratory control between the 1950s and 1960s [7], [8]. Early works in physiological modeling and analysis from the control system perspective are available in [9]–[11]. Significant progress was made during the last decade due to developments in technology, such as computation, physiological sensors, and high-speed data acquisition. Monographs that focus on various areas of physiology as well as from different scientific perspectives can be found in [1]–[3], [12], and [13].

The book under review presents a technique for applying optimal control theory and parameter estimation to the analysis of regulation processes in the cardiovascular and respiratory systems. Models introduced in this book are derived from the principles of the physiological mechanisms rather than descriptions of input-output relationships. Therefore, the state variables in these models usually possess physiological meaning, which is useful in broader applications in medicine.

**CONTENTS**

The target audience for this book includes researchers and graduate students from applied mathematics and control systems who are interested in applying control theory to cardiovascular and respiratory systems. The book is organized into five chapters and three appendices. Each chapter begins with an introduction to the key physiological concepts related to the topics in the chapter. Following the physiological concepts, a mathematical model representing the dynamics of the physiological system of interest is provided with a clear explanation. The model is then formulated as a control systems problem for analysis and parameter estimation. Clinical applications of the model are presented as examples. The appendices summarize control theories used in the analysis in each chapter.

The first chapter introduces the basic model of the cardiovascular system. This model is formulated as a linear-quadratic regulator problem to simulate the baroreceptor loop in response to a low-to-medium workload. A numerical algorithm is used to estimate the unknown parameters, and the accuracy of the model is demonstrated experimentally by using a bicycle ergometer. In Chapter 2, the respiratory control system is considered. Time delays are included in the model equations, representing the transport delays in the concentrations of partial pressures. The stability of the respiratory control system associated with the time delays is analyzed, and several clinically relevant applications are provided.

Chapter 3 integrates the models presented in the first two chapters to describe the interactions between the cardiovascular and respiratory systems. The state-dependent delays due to model integration and determination of the state-variable equilibria are discussed. Heart rate and ventilation rate are determined by minimizing a quadratic cost function of the state variables associated with the model. Applications of the cardiovascular-respiratory model in the analysis of congestive heart failure, the impact of gravity on cardiovascular function, and bleeding are presented.

Chapter 4 discusses the influence of venous function to the cardiovascular system control, including blood, plasma, and red blood cell volumes and their regulation. Application of the venous system model in hemodialysis is also discussed. Chapter 5 presents recent development in cardiovascular-respiratory system modeling and analysis as well as potential future clinical applications.

**COMMENTS**

The topics included in this book focus on the response of the physiological variables under steady-state equilibria. The responses of these variables involve much slower dynamics than the cardiac period or the breathing rate. For instance, the transition time of the cardiovascular system in response to work load changes is much longer than the beat-by-beat variation in the blood pressure and flow waveforms. The chemoregulation of ventilation is determined by the mean partial pressures of $O_2$ and $CO_2$ in the arterial blood. Therefore, the models introduced in this book ignore the pulsatile functions of the heart and lungs. For research in which the instantaneous pressures and flows are important, models that include the beating heart or the respiratory muscle function are given in [1].

The contents of this book are well organized. The authors did a wonderful job providing thorough descriptions of the models and analyses in the first three chapters. For readers who are knowledgeable in optimal control, the mathematical derivations should be easy to follow. However, understanding the physiological meaning of the model equations could be a challenge, even though the key physiological concepts are provided before introducing the models. For mathematicians and engineers who need more knowledge of cardiovascular and respiratory physiology, a textbook of human physiology such as [14] would be helpful. Using control system concepts in the analysis of blood volume control and the venous system in Chapter 4 is not described as clearly as it is in the first three chapters. A more detailed explanation regarding the application of the venous system model to hemodialysis would be helpful.
CONCLUSIONS
Cardiovascular and Respiratory Systems Modeling, Analysis, and Control provides sufficient information for applying control theories to analyze the physiological control mechanisms of the cardiovascular and respiratory systems. This text is a useful reference book for control researchers who wish to explore applications in medicine. Furthermore, the book can be used as a textbook for a graduate-level special topic course, although additional references in human physiology and optimal control systems might be needed.

REFERENCES

REVIEWER INFORMATION
Yih-Choung Yu has been a faculty member in the Electrical and Computer Engineering Department at Lafayette College since 2001, where he is currently an associate professor. He received the Ph.D. from the University of Pittsburgh in 1998. From 1998 to 2001 he was a principal engineer at CardioAssist, Inc., where he was involved with monitoring system design for a ventricular assist device. His research interests are in modeling, identification, and control for medical device development, as well as minimally invasive estimation of physiological functions. He is a Senior Member of the IEEE.

The theory is complemented by several examples and problems, some of which are direct applications of the theory to numerical cases, while others have a more theoretical flavor and may enhance the theorem-proving skills of hard-working students. The motivating examples are mainly classical, taken from mechanics, electrical circuits, and electromechanics, as well as ecology (Lotka-Volterra) models. I confess that, biased by my research interests, I would have liked to see some more catchy models, from systems biology for instance, a field of science currently undergoing a revolution because of the use of quantitative models (often precise differential equations) as an investigation tool to make predictions that are later validated or invalidated by experiments [1], [2].

Several black-and-white figures illustrate, when appropriate, definitions or ideas in an intuitive way. For instance, in the chapters devoted to stability analysis, Lyapunov functions are clearly exemplified, and intuition in terms of their level sets is provided.

BOOK ORGANIZATION AND COMMENTS
The first chapter of the book illustrates systems of differential equations in various fields and introduces the terminology needed to classify such models according to their dynamical features. Also, the notion of initial value problem is introduced.
Chapter 2 is devoted to the fundamental theory by proving results on the existence, uniqueness, and continuity of solutions and their differentiability with respect to initial conditions for nonautonomous systems, under fairly relaxed assumptions. To enhance readability, proofs are carried out for scalar nonautonomous systems, while their extension to the $n$-dimensional case is left to the reader, who is provided with some elementary background material on vector analysis. The main existence proof relies on approximate solutions and the Arzela-Ascoli lemma, which is stated and derived in the same chapter, for the sake of completeness. In the problem section, the contraction mapping theorem is suggested as an alternative tool for the proof of the same result.

The theory developed in Chapter 2 is specialized to the linear case in Chapter 3. A large amount of algebra background is first recalled, in a self-contained and concise way, starting from the notion of matrix, linear independence, and eigenvalues, all the way to the Jordan canonical form, which is the main tool for understanding the structure of solutions of linear autonomous systems. The treatment is carried out in the time domain, both for time-varying and time-invariant systems, leading to the introduction of the matrix exponential. A useful topic, which is usually not treated in a basic dynamical systems book, is a section devoted to oscillation theory for second-order, time-varying linear systems. Indeed, another feature of the book, besides the stability of equilibria, is the study of oscillatory behavior.

Motivated by the study of partial differential equations, boundary value problems for ODEs are introduced in Chapter 4. The material here is not completely self-contained, which, as far as I could see, is not essential for subsequent chapters.

Chapter 5 deals with stability questions for equilibria of systems of differential equations. Since this textbook is about ODEs, the theory develops notions of stability only with respect to initial conditions. No exogenous disturbances or signals are introduced. The classical Lyapunov methods are presented, including both linearization techniques and Lyapunov functions. Algebraic criteria for testing stability of linear systems, such as the Routh-Hurwitz criterion, are also stated without proof. Definitions are provided in an $\varepsilon - \delta$ style, without using comparison functions, such as $K_{\infty}$ or $KL$ functions, which are by now standard terminology in nonlinear control textbooks such as [3]. We feel that this limitation is a little old fashioned and that it would be useful to provide connections with these recent tools, which, in my opinion, convey a more direct insight into the definitions, while allowing for fairly deep manipulations once a few crucial properties of those functions are established. As a plus, it is worth mentioning that converse Lyapunov theorems are discussed, including Zubov’s theorem, which is not always the case in ODE textbooks. Absolute stability is also studied by means of the Popov criterion and Lur’e-Postnikov Lyapunov functions.

The last three chapters provide more advanced topics including, among others, the existence of stable and unstable manifolds, stability of periodic solutions, the Poincaré-Bendixson theorem for planar systems, Andronov-Hopf bifurcations, and even an interesting nonexistence result (which, by the way, I was not aware of), relating a lower bound for the minimal period of oscillation of a general $n$-dimensional autonomous system to its Lipschitz constant.

As the preface states, “there is more than enough material in this text for use as a one-semester or a two-quarter course,” condensed in just about 350 pages. Overall, it is a nicely written book, which, as testified by the selection of topics, appears to be especially suitable for a control engineering curriculum.

REFERENCES

AUTHOR INFORMATION
David Angeli graduated in control engineering in 2000 from the University of Florence, Italy, where he is currently an associate professor in the Department of Systems and Computer Science. He is also a senior lecturer at the Imperial College of London. His research interests include stability of nonlinear systems, constrained and adaptive control, systems biology, and chemical reaction networks. He is the author of more than 40 publications in peer-reviewed journals and an associate editor for IEEE Transactions on Automatic Control and the IMA Journal of Mathematical Control and Information.