Modeling and Control of a Waste-to-Energy Plant
Waste-Bed Temperature Regulation
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Municipal and industrial waste has been treated in incineration plants since the beginning of the 20th century, mainly to reduce the landfill volume [1]. The incineration process transforms environmentally hazardous components of waste, such as aromatic hydrocarbons and organic solvents, into harmless compounds. Additionally, incineration plants convert the released combustion heat into steam and electricity. The steam can be used for industrial processes or district heating, and the electricity is usually fed into the national grid [1]. In 1999, the European parliament passed a law that requires every country in the European Union (EU) to stop disposing of municipal waste without treatment after 2005, thus making incineration the only feasible treatment [2].

For bulky solids, highly combustible wastes, or wastes with a high water content, incineration offers a significant volume reduction and provides detoxification for combustible carcinogens, pathologically contaminated materials, toxic organic compounds, and biologically active materials that can affect sewage-treatment plants. Moreover, incineration mitigates the environmental impact of organic material that can leach from landfills or create an odor nuisance. In addition, the impact of the greenhouse gases generated by incinerating solid waste is less than that of the methane generated by landfiling operations. Additionally, because of strict air pollution emission requirements applicable to municipal refuse incinerators whose waste-to-energy facilities produce electricity, pollutant air emissions per kilowatt-hour produced, though still present and potentially posing a health hazard, are less than that generated by coal- and oil-burning utility plants. Energy recovery is feasible when large quantities of waste are available and reliable markets for by-product fuel, steam, or electricity are nearby. Incineration forms oxides and glassy, sintered residues that are insoluble. This residue can then be disposed of in a landfill without biodegradation of organic material leading to either subsidence and gas formation, which disrupts the landfill-capping structures, or leaching, with potential contamination of underground water [1].

The waste incineration process is sensitive to the complex chemical processes and unsteady fuel qualities of the waste, such as water content and heating value, which are not available online. Consequently, the demand for reliable control techniques is high and has increased significantly since 2005, especially for combustion [3], [4]. This article describes the incineration process and its mathematical modeling. The model [5], which focuses on the chemical and thermodynamic aspects of the process, is the base for the development of an advanced controller that uses input/output linearization and extremum seeking [6], [7].

WASTE INCINERATION PROCESS

We now describe the main thermochemical processes taking place in an incineration plant with moving grates, described in “How a Municipal Solid Waste Incineration Plant Works.” These processes include drying, pyrolysis, gasification, char combustion, and ash sintering. During steady-state operations, these processes simultaneously take place in consecutive zones of the grate. The equations used in the model can be found in “State Equations.”

Drying

The drying zone is an important component of the waste incineration process since it influences the location of the combustion area on the grate. The temperature distribution in the furnace, residence time of the flue gases, and level of the material stress of the combustion bed grate depend on the onset timing of the combustion. If the onset time of the combustion is too soon or too late, then the temperature distribution in the furnace may not be homogeneous, affecting the drying process, which is strictly connected with the heat transfer in the waste bed. In a quasi-homogeneous bulk streamed by a fluid, such as the waste bed on the grate, the heat transfer process is given by the sum of the convective heat transfer \( Q_c \), between the gas and particles, the incoming radiation \( Q_{rad} \) from the combustion zone, and the heat conduction \( Q_l \) at the contact point between two particles. The heat transfer, and therefore the drying, is then a function of the fluid velocity and its characteristics, such as heat capacity, density, heat conductivity, and viscosity [8], [9].
Pyrolysis and Gasification

When all the water in the waste has evaporated, the temperature of the waste increases. The gasification of volatiles, that is, combustion under oxygen defect, starts as soon as the temperature of the waste reaches a certain value, and shortly thereafter pyrolysis starts. Pyrolysis occurs in the absence of oxygen when higher hydrocarbons of the form C\textsubscript{x}H\textsubscript{y}O\textsubscript{z} are cracked into smaller gaseous species, such as CH\textsubscript{4}, CO, and H\textsubscript{2} at high temperature levels. Therefore, the primary airflow through the waste bed is maintained below stoichiometric levels. To start pyrolysis, waste is heated up to a temperature at which the oxygen reacts with some of the volatile hydrocarbons and solid components in exothermic reactions; see “Process Equations.” The heat release of these reactions, the most common being the water-gas shift, supplies the energy to run the pyrolysis, but since most of the oxygen is consumed by the initial reaction there is almost nothing left to oxidize the newly gasified hydrocarbons. Thus, a mixture of gasified hydrocarbons, water from the reaction of oxygen and hydrogen, and the remaining nitrogen, leaves the waste bed and becomes the fuel gas for the homogeneous secondary combustion above the waste bed. The final composition of the pyrolysis gases as well as the volatilization ratio depend on the composition and temperature of the waste along with the residence time of the hot gas in the waste bed.

Char Combustion

After pyrolysis is completed, the residual carbon, called char, starts combusting. Char combustion is described relative to the combustion of a carbon sphere since all pyrolyzing solids, once completely charred, are attacked by oxygen in much the same manner as carbon [10]. The combustion of a carbon particle is accompanied by a sufficiently high surface temperature to become incandescent. Oxygen diffuses from the free stream to the surfaces where it reacts directly with the solid to release a large quantity of heat. Much of the heat is lost to the surroundings through radiation. The burning rate of simple solids depends on the rate at which the oxygen diffuses to the fuel surface [10]. Heterogeneous reactions involve five sequential steps:

1) Oxygen diffuses to the fuel surface.
2) Diffused oxygen is absorbed by the surface.
3) Absorbed oxygen reacts with the solid to form absorbed products.
4) Absorbed products are desorbed from the surface.
5) Desorbed products diffuse away from the surface.
These steps occur in series, with the slowest step determining the burning rate. When the temperature is low, the particle is small, and the flow around it is feeble; step 3 thus becomes much slower than steps 1 and 5. In this case, the burning rate is determined by chemical kinetics, and therefore the process is kinetically controlled by the kinetic burning rate $k_{\text{chem}}$. On the other hand, when the particle and flow velocity are large and the temperature is high, step 3 is known to be much faster than steps 1 and 5. In this situation the burning rate is controlled by the diffusion rate of oxygen to the particle, that is, the diffusional burning rate $k_{\text{diff}}$. In the diffusional controlled regime, the burning rate has a weak dependence on the temperature and a strong dependence on the particle size. The overall burning rate $k_{\text{eff}}$ is given by the sum of the two burning rates.

Ash Burnout

Municipal solid waste includes inert materials that cannot be destroyed in the combustion process. Since the incineration process is inherently imperfect [1], some potentially combustible materials are dried, heated, and carbonized, but the desired next step, namely, gasification or burnout of the char, is not achieved. Furthermore, some material falls between the grate bars and passes through the hot combustion environment substantially unburned. These three components, called, respectively, inerts, uncombusted, and siftings, comprise the ash, which is the inevitable residue of municipal solid-waste incineration operations.

Municipal solid ash is usually characterized as bottom ash or fly ash. Bottom ash is the ash that falls from the end of the grates combined with the siftings that fall through the grates. The fine ash that becomes airborne in
State Equations

The equations modeling the incineration process are of the form

\[ \dot{x} = f(x) + g(x)u, \quad (S1) \]
\[ y_1 = h_1(x, u), \quad (S2) \]
\[ y_2 = h_2(u). \quad (S3) \]

The states, as described in the section “Process Modeling,” are the mass and temperature in the lower pile and the temperatures in the upper pile, the gas flame, and the flue gases. For consistency, the naming convention in “State Equations” is used. The time derivative of the waste is the vector \( M_{LL} \in \mathbb{R}^4 \), whose components are the changes in water, volatiles, fixed char, and inerts. With reference to the naming convention used in the section “Waste Incineration Process” and the equations in “Process Equations,” the changes due to the relevant thermodynamic processes can be written as

\[ d_{\text{thermal}} = \left[-\frac{n_0 A_{k_{\text{evap}}}}{P_v} \left(\frac{P_v}{1} - \frac{P_P}{P_v}\right) \epsilon_i M_{LL}ight], \quad (S4) \]

where \( \epsilon_i \) is the transposed unit vector with index i, and \( k_{ij} \) is the pyrolysis pre-exponential factor.

The overall mass variation is therefore given by

\[ d_{\text{waste}} = d_{\text{thermal}} + V_{\text{pp}} k_{v} M_{LL}. \quad (S5) \]

The ash bunker, which collects the ash falling from the last pile, is modeled as an infinite sink modeled by the equation

\[ \dot{x}_{AB} = -V_{\text{pp}} k_{v} M_{LL}. \quad (S6) \]

The global waste-mass balance, including the ash bunker, yields

\[ \dot{m} = \sum_{i=1}^{n} V_{g} k_{v} M_{LL} - V_{g} k_{v} M_{LL} - d_{\text{waste}} + V_{g} k_{v} M_{LL} = f_{\text{w}} - \sum_{i=1}^{n} d_{\text{waste}}. \quad (S7) \]

where \( f_{\text{w}} \in \mathbb{R}^4 \) is the incoming waste from the feeding system, and \( n \) is the number of piles.

In the upper layer the only state is the temperature \( T_\text{u} \in \mathbb{R} \), whose variation \( T_\text{u} \) is a function of the conductive heat transfer and radiative transfer with the flame. The conductive heat transfer is a stabilizing element, since it tends to equalize the temperatures in the upper and lower layer as shown in the section “Waste Incineration Process.” The gas cloud has one state, namely, the gas temperature \( T_\text{u} \in \mathbb{R} \), which varies because of the radiative heat transfer with the burning bed, the primary and secondary combustion processes, and the incoming energy from the secondary air.

As described in the section “Waste Incineration Process,” each thermochemical process is bound to a heat release or heat absorption. This heat flow is proportional to the mass change, with constant \( k_{v} \). \( c_{p} \) indicates the heat capacity for the various materials, that is, waste, air, and combustion gases, as given by the equations

\[ d_{\text{a}} = k_{d} d_{\text{v}} \in \mathbb{R}^4, \]
\[ d_{\text{s}} = k_{v} [T_{\text{OC}} - T_{\text{s}}] \in \mathbb{R}, \]

the primary chamber is instead called fly ash. Fly ash either settles in the ducts and devices of the incinerator or continues to the air-pollution control system. The fly ash also includes refuse constituents, which volatize in the high-temperature zones of the furnace and subsequently condense on the small-diameter particles that present a large surface area, called particulate. These constituents may include heavy metals, such as lead, cadmium, copper, zinc, and high-molecular-weight polycyclic aromatic hydrocarbons with potentially negative health effects. In the final part of the bed, the grate bars move much more slowly than in the front part, and the bed starts to rise again. The residence time in the last zone of the furnace must be sufficiently long to ensure the loss of incandescence for the ash. The ash is quenched and cooled with water before it is transported to the slag bunker by means of a conveyor belt. The heat loss in the process, which is caused by the ash, has to be considered in the overall heat balance of the process. It is legally required in the EU [2] that the mass fraction of unburned carbon in the ash not exceed 2% of the total mass. In addition to the legal nature of this constraint, the loss in chemical potential due to unburned carbon components must also be considered in the heat balance.

Secondary Gas Reactions

The combustible gases generated during the gasification and pyrolysis phase leave the waste bed with the primary air stream. As soon as this mixture of hydrocarbons, water vapor, nitrogen, and remaining oxygen leaves the waste-bed surface, it is ignited and burns in a homogeneous gas-phase reaction. To ensure complete combustion, secondary air is injected into the combustion zone to maintain fuel-lean conditions. Thus, the stoichiometric coefficient of the secondary combustion zone is given by the amount of excess air. The oxygen needed for complete combustion is calculated from the total amount of air sent to the furnace and the remaining oxygen at the furnace exit. Typically, the excess oxygen lies between 6% and 8%. The compounds that react with oxygen or with each other in the secondary combustion zone include water vapor from the drying and previous reactions, methane, ethane, benzene, and carbon monoxide.
Production can also be optimized. Given the economic much as possible without reducing throughput. The steam able to decrease the amount of air pollutant emissions as of waste that can be treated. At the same time, it is desir- tion plant allow, in the first place, an increase in the amount of steam that can be treated. At the same time, it is desirable to decrease the amount of air pollutant emissions as much as possible without reducing throughput. The steam production can also be optimized. Given the economic value of steam and electricity, there is a tradeoff between maximising throughput and maximising steam production. Moreover, it is necessary to guarantee that the amount of carbon in the remaining ash does not exceed the limits set by environmental laws. Ash with more carbon than legally allowed for common waste deposits must be disposed in hazardous waste deposits, thus increasing the operational costs of the plant.

Control and optimization of the waste combustion depend on the plant and grate configurations. The most common loops are the oxygen fraction in the flue gases controlled by varying the primary air flow or the total air flow, the steam flow rate controlled by varying the waste-feed rate, and the bed height maintained within a given range by adjusting the grate speed. The waste-feed rate is normally controlled indirectly by an additional control loop by varying the feeder speed and its length [11]. The secondary air is either controlled in an open-loop mode or given as a fraction of the total air to control the flue gas temperature in the combustion chamber. The bed height is estimated from the underbed air pressure. The bed height in the main combustion zone, which governs the flame position and waste-bed temperature, is controlled indirectly by maintaining an
Process Equations

The main processes taking place in an incineration plant are drying, pyrolysis, gasification, char combustion, secondary gas combustion, and ash sintering. These processes, with the exclusion of the ash sintering, are modeled as follows:

**Drying**

\[ Q = hA(T_g - T_s) \]
\[ Q = \sum q_i \sigma (T_i^4 - T_f^4) / (1/\varepsilon_i + 1/\varepsilon_f - 1) \]
\[ Q = -\Delta T / \Delta x \]
\[ m_{\text{evap}} = -n_p A_s k_{\text{evap}} (P_{\text{vap}}(T_p) - PP) \]
\[ k_{\text{evap}} = ShD_{\text{H}_2O} / d_p \]

**Pyrolysis**

\[ k_{\text{vol}} = k_{\text{vol}} e^{-E/RT} \]
\[ m_{\text{vol}} = k_{\text{vol}} (m - m_0) \]

**Gasification**

\[ CO + H_2O \rightarrow CO_2 + H_2 \]

**CHAR COMBUSTION**

\[ k_{\text{eff}} = (M_c Sh \Phi D_\phi / (Rd_p T_m)) (T / T_s)^{1.75} \]
\[ k_{\text{chem}} = A_e \theta e^{-E_\phi/RT} \]
\[ k_{\text{eff}} = 1/(1/k_{\text{eff}} + 1/k_{\text{chem}}) \]
\[ m = -n_p A_s k_{\text{eff}} p_{\text{in}} \]

**SECONDARY GAS COMBUSTION**

\[ 2CO + O_2 \rightarrow 2CO_2 \]
\[ CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O \]
\[ C_2H_6 + 3.5O_2 \rightarrow 2O_2 + 3H_2O \]
\[ N_2 + O_2 \rightarrow 2NO \]

The activation energy value \( E \), the pre-exponential factors \( A_e \) and \( k_{\text{vol}} \), and the convective mass-transfer coefficient \( k_{\text{evap}} \) are taken from the literature [16]–[18]. The term \( Sh \) denotes Sheffield number, \( d_p \) is the particle diameter, \( n_p \) is the number of particles, \( A_s \) is the particle surface area, and \( P_{\text{vap}} \) and PP are, respectively, the saturated vapor pressure at temperature \( T_p \) and the vapor partial pressure in the surrounding gas phase.

Adequate underbed air pressure. These control actions are typically performed manually by the operators, although there is high potential for process automation. Manual operation is used in almost every waste incineration plant in Europe; however, significant operational disadvantages occur due to an insufficient degree of automation. The standard operation procedures demand a high degree of attention and fast decision making by the operators in case of emergency. This fast response is not always possible since the majority of sensor data presented to the operator’s control panel have a delay of several seconds and are not measured directly at the relevant plant location. In addition, some fundamental data, such as water content and heating value of the waste, are not available.

The use of infrared thermography opens new scenarios for waste incineration. Infrared thermography makes it possible for the operator to determine the position of the flame and react appropriately. It is also possible, by statistical evaluation of the infrared picture, to determine the width of the combustion zone and derive from it information on the asymmetric location of the fire or secondary combustion zones. This information can be used to improve the performance of standard controllers [4].

A conventional control approach is to use decoupled proportional-integral-derivative (PID) loops [11] to control the air flow rate, waste-feed rate, and grate speed.

**PROCESS MODELING**

To generate a suitable model describing various plants with individual process concepts without the need to change major structural components, a physics-based approach is used; see Figure 1. The various processes and components are described using thermodynamic and chemical laws, avoiding the use of lookup tables based on measured data. All values and constants are taken from [1] and [11].

The simplest model unit is called a pile [5]. A pile is a volume of waste on a grate zone. Each pile is assumed to be homogeneous with regard to temperature and composition. Additionally, the waste-bed properties are considered to be constant over the grate width, meaning that the waste properties vary along the direction \( x \) of the waste movement as well as in the vertical axis \( y \), but are constant across the width \( z \). An upper layer is modeled, representing the thin layer of waste on top of the pile, subject to radiation from the gas cloud above; see Figure 2. As the temperature of the upper layer rises, heat is transferred by convection into the lower layer. Each pile has inputs from the primary air flow \( f_p \) and its temperature \( T_p \), the grate speed \( V_g \), and the incoming waste feed rate \( f_w \) and its temperature \( T_w \); see “State Equations.” For the first pile, the incoming waste is from the feeder, whereas, for the other piles, the incoming waste comes from the previous pile. The states for each pile are the lower layer mass \( M_{LL} \) in its four components, the lower layer temperature \( T_L \), and the upper layer temperature; see “State Equations.”

The secondary air flow \( f_{\phi} \) and its temperature \( T_{\phi} \) and the energy input \( Q \) from the auxiliary burners are inputs to the overall model, while the gas-cloud temperature \( T_p \), the flue-gas temperature \( T_{\text{CG}} \) at boiler exit, and the steam flow...
rate $S$ are states. Both inputs and states are also considered in the model. The measured outputs are the steam flow rate $S$, the residual oxygen $O_2$, the flame temperature $T_f$, the flue-gas flow $f_g$, and its temperature $T_{fg}$, and the height $h_w$ of the waste pile. Mass and energy are conserved in the equations given in “State Equations.”

We consider a model with four piles laid in series, creating a module defined as a grate; see Figure 3. Primary air is blown under each pile, and each pile can be moved with a grate speed $V_g$. Mass transfer between adjacent piles occurs only in the positive $x$ direction, that is, the backward thrust effect is ignored. On the first pile the waste is dried, while on the second and third pile, pyrolysis and glowing char combustion occur. The fourth pile is used mainly for ash burnout. The gases thus produced are combusted in the combustion chamber above the bed where secondary air is injected. In the case of a waste to energy plant, that is, an incineration plant with energy recovery, the combustion chamber is a standard boiler; see “How a Municipal Solid Waste Incineration Plant Works.”

Local stability of an operating point of the system is proven by linearization. Moreover, the linearized system is observable and controllable. Some input/output transfer functions have zeros in the right-half plane, making the linearized system nonminimum phase. Additionally, the nonminimum-phase behavior for such input/output couples is visible when observing the form of the step response.

**NONLINEAR CONTROL**

We assume that a flame temperature measurement, such as an infrared camera or a camera in the visible range, provides online measurements of the waste-bed temperature. Maintaining a stable combustion zone ensures a correct temperature profile on the grates, an optimal heat release, and correct flue-gas residence time. Moreover, a stable profile ensures that the waste is completely burned before falling into the ash pit, therefore reducing the heat losses and improving the ash quality. A correct combustion profile can be obtained by a combination of the control variables, waste-feed rate, grate speed, and primary and secondary air.

We consider an approach that relies on input/output linearization [6], [12] and extremum seeking [7]. The goal is to define a set of values for the waste-bed temperature so as to maintain a steam setpoint and then maximize it. At the same time, additional outputs, such as the residual oxygen in the flue gas or the combustion chamber temperature, must be kept within specified values. Stability is achieved using the approach described in [6]. Each pile is described according to the equations in “State Equations,” and the temperature of each pile is set to be equal to a reference value. Additionally, we require that the oxygen $O_2$ and waste-bed height $h_w$ for the second pile reach given reference values. The waste-bed height is calculated as a function of the waste density $\rho_{\text{waste}}$, waste composition $\alpha$, waste mass $x_{\text{LL}}$, and grate zone area $A$.

The vector relative degree [6] for this system is $r = [1 \ 1 \ 1 \ 1 \ 1]$, implying that the first time-derivatives of the output signals are explicit functions of the input signals. As a result, there exists an input $u$ such that, for each constant output reference value $y^*$, the equation

$$\dot{y} = \frac{\partial h}{\partial x} (f(x) + g(x)u) = -k(y - y^*), \quad (1)$$

with $k > 0$, has a solution. This solution is subject to constraints because of actuator limitations. The constraints define, for instance, the total air flow range, the maximal waste-feed rate, and maximal grate speed, which are expressed in the form $Au \leq B$. Therefore, the control
Waste Composition

According to [2], the expression “mixed municipal waste” means waste from households as well as commercial, industrial, and institutional waste that, because of its nature and composition, is similar to waste from households but excludes waste defined as hazardous [15]. Household waste is characterized by a strong heterogeneity; to better analyze its composition it is further subdivided into four subgroups.

1) Material that burns or can be composted, food and kitchen leftovers, paper and shredded cartons, textiles without plastic, and yard waste.
2) Material that can only be combusted, such as wood, unshredded carton, textiles with plastic components, rubber, leather, and cotton waste.
3) Material that cannot be burned or composted, including glass, porcelain, stones, and metals.
4) Fine waste (less than 8-mm diameter), ash, sand, and organic material of small dimension.

From the stream analysis, which means from the qualitative analysis in terms of the above-mentioned groups, it is possible to define the waste fractions of water, inerts, and combustibles; see Figure S2(a). These fractions define the proximate analysis. A more precise description is given by the ultimate analysis, also known as the elemental analysis, which gives the composition in terms of the main chemical components C, H, O, N, and S, that is, carbon, hydrogen, nitrogen, oxygen, and sulphur; see Figure S2(b). The ability of waste to be used as a combustible is given by the lower heating value (LHV). Assuming as a reference threshold a typical average heating value for wood, \( LHV_{\text{Wood}} = 19,000 \text{ kJ/kg} \), municipal waste finds itself usually in the range \( LHV_{\text{Waste}} = 8000–12,000 \text{ kJ/kg} \) [16]. This calorific value makes it a suitable candidate for incineration as well as for heat and energy recovery.

Simulation at Steady State

The first test consists of fixing the reference values for the waste-bed temperature and checking whether the implemented algorithm succeeds in achieving these steady-state conditions. The waste composition \( \alpha \in \mathbb{R}^4 \) is assumed to be constant; the components of the vector are, respectively, water, char, volatiles, and inerts. If the reference for the waste temperature varies, then the step response is fast, namely, about 3 min for a temperature change of 50 K; see Figure 4.

Variable Waste Composition

Since waste composition is highly variable and cannot be measured online (see “Waste Composition”), any process control strategy for waste-incineration plants must be robust against variable waste composition. The waste composition \( \alpha \) varies dynamically during the simulation. The water and combustible material fractions, namely, fixed carbon and volatiles, are normally distributed random numbers, with a given mean and variance, while the ash fraction is obtained by normalization. In practice it would be reasonable to assume a waste composition change every 20–30 min. This value is compatible with the amount of waste grabbed by the fresh feed crane, which is on the order
of 2–3 tons, for a middle-sized plant. Considering a deep enough waste bunker and a skilled crane operator, it is reasonable to expect that the changes over this period are modest, while the composition can vary significantly on a daily, weekly, or seasonal basis. By making a step change in the waste composition, the time constant of the process can be determined, which turns out to be about a half hour; see Figure 5.

**Sensitivity of the Pile Temperature**

Once proven that it is possible to reach different steady states, we wish to determine the steady states that maximize the steam production. For varying setpoint values and constant waste composition, the steam production shows a maximum. The analysis of the results has to limit itself to the temperature range where the setpoints on temperature and oxygen are achieved; steam values for which the setpoints are not achieved are discarded. Since the control variables are constrained, some setpoints cannot be reached. The constraints are nonetheless left unvaried because the sensitivity to the waste-bed temperature must be tested while remaining within a realistic plant configuration. As shown in Figure 6, for the tested compositions it is possible to identify a waste-bed temperature range containing a relative maximum for the steam flow rate.

The next step is to find a way to determine the steady-state temperature setpoints that maximize the steam production, even when the waste composition varies. At the same time the constraints and conditions on the remaining states must be respected.

**EXTREMUM SEEKING ANALYSIS**

As shown above, it is possible to stabilize an operating point for some sets of temperature and oxygen values, while the steam flow rate presents a maximum for at least some pairs $T_2, T_3$ and constant oxygen. These maxima vary with waste composition. The next problem is to determine a reference.
pair $T_2, T_3$ that maximizes the steam as the waste composition varies.

To determine $T_2, T_3$, we use the extremum-seeking approach [13]; see figures 7 and 8. This approach does not require knowledge of the equilibrium map, but only the existence of a maximum or minimum for the output and the fact that the nonlinear system can be stabilized around each equilibrium by a local feedback controller. We use the extremum seeking as an external loop providing setpoints; the input/output linearization stabilizes the system around them. In this way, maximization or minimization of a specified output, such as the steam flow rate, is achieved without any knowledge of the waste composition. Given the constraints on the inputs, it is not always possible to maximize the steam production directly. Instead, a convex cost function $L$ of the error between the actual steam flow rate $S$ and two estimated minimal and maximal steam values, defined as

$$L = S - m, \quad S < m,$$

$$L = \left[-\frac{qS}{m-M} + \frac{qm}{m-M}\right], \quad m \leq S \leq M,$$

$$L = -S + M + q, \quad S > M,$$

is minimized. The minimal value $m$ and the maximal value $M$ are defined by the user according to the average waste composition, while $q$ is a tuning parameter.

The results of the first test, run with constant waste, are presented in Figure 9. The results show that the extremum-seeking routine calculates new setpoint values for the

**FIGURE 7** Schematic of the control solution. The extremum-seeking routine minimizes a convex cost function of the deviation between the steam flow $y$ and a given steam flow range $[M, m]$, providing a pair of target values for the temperature in the second and third pile. The constrained input/output linearization calculates an input vector $u$ such that the temperature targets $\theta$ are reached.

**FIGURE 8** Schematic of the extremum-seeking solution. The process output is the steam flow rate $y$, $M$ and $m$ are the target range limits for the steam flow rate, $d$ is a disturbance, and $k_1, k_2, k_3$, and $k_4$ are positive scalars. While the solution described in [13] uses derivatives, a constant time delay of 6 time units is used here to avoid reacting to the short-term process noise. The extremum-seeking routine minimizes a convex cost function, defined in (4), providing a pair of target values $\theta$ for the temperature in the second and third waste-bed piles.

**FIGURE 9** Steam maximization. This plot shows the steam flow rate, obtained in the following four cases. The cyan curve is obtained by setting the second pile temperature to $T_2 = 900$ K and the third pile temperature to $T_3 = 1350$ K; the red curve is obtained by setting $T_2 = 900$ K, while leaving $T_3$ free to vary; the blue curve is obtained by setting $T_3 = 1350$ K, while leaving $T_2$ free to vary; and the green curve is obtained by leaving $T_2$ and $T_3$ free to vary. The target steam production is constrained to lie in the interval $[37.5 \text{ tons}/\text{h}, 38.5 \text{ tons}/\text{h}].$

**FIGURE 10** Extremum seeking with variable waste. The cyan curve is obtained by setting the second pile temperature to $T_2 = 900$ K, and the third pile temperature to $T_3 = 1350$ K, while the blue curve is obtained by leaving $T_2$ and $T_3$ free to vary. The target steam production is constrained to lie in the interval $[37.5 \text{ tons}/\text{h}, 38.5 \text{ tons}/\text{h}].$ Since, according to Figure 6, we expect to find a maximal value for the steam flow in this range. The waste composition varies with a time constant of 30 min and is smoothed over 2 h.
The waste incineration process is sensitive to the complex chemical processes and unsteady fuel qualities of the waste, such as water content and heating value, which are not available online.