How Good Is Your Gyro?

In this issue of IEEE Control Systems Magazine, we invite Mohinder Grewal and Angus Andrews to respond to an inquiry about gyro specifications. Readers are encouraged to submit technical questions, which will be directed to experts in the field. Please write to us about any topic, problem, or question relating to control-system technology.

Q: In my design course in aerospace engineering, my group is building an unmanned aerial vehicle (UAV), which means the aircraft must fly on its own. My task is to choose a gyro for the aircraft to measure angular rates about the aircraft axes. The problem is that when I look at the spec sheet for gyros there are many numbers that I do not know how to interpret. Can someone at IEEE Control Systems Magazine please explain gyro specifications?

Mohinder and Angus: We're happy to answer your question, which is really a great question. Rate gyroscopes, which I will call “gyros,” are sensors that measure angular (rotational) rates with respect to an inertial frame of reference. An inertial reference frame is a coordinate frame in which Newton’s laws of motion are valid.

Gyros are used in many applications, including inertial navigation, robotics, and automobiles. In the inertial navigation system (INS) of a vehicle, the equations of motion are numerically integrated using the outputs of gyros and accelerometers to estimate the vehicle’s attitude, position, and velocity. Because inertial navigation has co-evolved with the supporting gyro technology, the methodologies for relating gyro performance to mission requirements are fairly well established. These methodologies may not be directly transferable to other applications, such as robotics, automotive systems, or pointing stabilization of a telescope. Instead of navigation, these applications use gyros for stabilization. However, the same gyro error models and specification methods used for inertial navigation can be used in stabilization applications.

Before defining the gyro error model and its specifications, we need to define gyro classifications and types. Gyros can be classified according to the physical phenomena involved in their operation. The earliest types of gyros used spinning wheels and the law of conservation of angular momentum. These devices have been redesigned in several ways to reduce angular drift errors due to bearing torques, such as “floated” designs, electrostatic suspension to reduce bearing torques, and dynamically tuned flexible structures to decouple bearing forces from gyro torques. More recent designs use sensing mechanisms based on alternative physical phenomena, such as...

TABLE 1 The 13 types of gyros can be sorted according to their general usage categories, which are strategic, navigational, tactical, and consumer.

<table>
<thead>
<tr>
<th>Gyro Type</th>
<th>General Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single degree of freedom (DOF) floated</td>
<td>✔</td>
</tr>
<tr>
<td>Two DOF floated</td>
<td>✔</td>
</tr>
<tr>
<td>Dry tuned gyro or dynamically tuned gyro (DTG)</td>
<td>✔</td>
</tr>
<tr>
<td>Electrostatically suspended gyro (ESG)</td>
<td>✔ ✔</td>
</tr>
<tr>
<td>Ring laser gyro (RLG)</td>
<td>✔ ✔</td>
</tr>
<tr>
<td>Zero lock gyro (ZLG)</td>
<td>✔ ✔</td>
</tr>
<tr>
<td>Interferometric fiber optic gyro (IFOG)</td>
<td>✔</td>
</tr>
<tr>
<td>Hemispherical resonator gyro (HRG)</td>
<td>✔</td>
</tr>
<tr>
<td>Quartz tuning fork gyro</td>
<td>✔ ✔</td>
</tr>
<tr>
<td>Silicon micromachined vibrational</td>
<td>✔ ✔</td>
</tr>
<tr>
<td>Vibrating wine glass sensor</td>
<td>✔</td>
</tr>
<tr>
<td>Vibrating disc sensor</td>
<td>✔</td>
</tr>
<tr>
<td>Nuclear magnetic resonance gyro</td>
<td>✔</td>
</tr>
</tbody>
</table>
vibrating quartz or silicon elements, nuclear magnetic resonance (NMR), or optical phenomena, as in the case of the ring laser gyro (RLG) and the fiber optic gyro (FOG).

Table 1 shows 13 types of gyros, sorted according to their suitability for general usage categories defined as strategic (very high performance navigation), navigational (medium-accuracy navigation), tactical (low-accuracy navigation), or consumer (non-navigational applications, for example, robots or the Hubble Telescope).

Once a gyro is designed, its sources of error need to be understood and characterized with respect to influences such as temperature and gravity. The parameters of interest are accuracy, repeatability, size, cost, weight, maintainability, and reliability [1]–[3].

All gyro errors can be broken down into various categories, including constant errors that are predictable or repeatable from turn-on to turn-on; predictable errors due to temperature; errors that are unpredictable from turn-on to turn-on but remain constant after each turn-on; and errors that vary randomly after each turn-on.

Mathematical models of gyro performance are used throughout the INS development cycle to meet specified performance metrics. Since extreme performance requirements for inertial sensors cannot always be met within manufacturing tolerances, these same models can be used to calibrate and compensate for fixed errors such as input/output scale-factor or bias (offset) variations. Bias error is defined as any nonzero output when the input is zero. Scale-factor error (b) often results from aging or manufacturing tolerances. Nonlinearity (c) is present in most sensors to some degree. Scale-factor sign asymmetry (d) is often from mismatched push-pull amplifiers. A deadzone (e) is usually due to mechanical stiction or lock-in (for example, in a ring laser gyro). Quantization error (f) is inherent in all digitized systems. Quantization error may not be zero mean when the input is held constant.

In addition to mathematical models used throughout the INS development cycle, error models are used in global navigation satellite system (GNSS)/INS integration to determine the optimal weighting (input gain) in combining GNSS and INS navigation data. Gyro sensor models are used in GNSS/INS integration to recalibrate the INS continuously while GNSS data are available. The latter approach allows the INS to operate more accurately during periods of GNSS signal outage.

There are numerous types of error models that address the various errors. Zero-mean random error models are used in Kalman filtering. White sensor noise, which is usually lumped together under “electronic noise,” may come from power supplies, intrinsic noise in semiconductor devices, or from quantization errors in digitization. Exponentially correlated noise, such as the temperature sensitivity of sensor bias, often looks like a time-varying additive noise source driven by external ambient temperature variations or by internal heat distribution variations.

Random-walk sensor errors are characterized by variances that grow linearly with time as well as power spectral densities that fall off at 1/frequency², that is, 20 dB per decade. Specifications are sometimes given for random-walk noise in gyro sensors, but mostly for the integral of the outputs, and not in the outputs themselves. For example, the “angle random walk” from a rate gyroscope is equivalent to white noise in the angular rate outputs. In similar fashion, the integral of white noise in accelerometer outputs is equivalent to “velocity random walk.”
The random-walk error model has the form
\[
X_k = X_{k-1} + \tilde{w}_{k-1},
\]
\[
\sigma_k^2 = E(X_k) = \sigma_{k-1}^2 + E(\tilde{w}_{k-1}^2).
\]

For time-invariant systems, we have
\[
\sigma_k^2 = \sigma_0^2 + kQ_{sw},
\]
where
\[
Q_{sw} \equiv E(\tilde{w}_0^2).
\]
The value of \(Q_{sw}\) is in units of squared-error per discrete time step \(\Delta t\). Random-walk error sources are usually specified in terms of standard deviations, that is, error units per square root of the time unit. Gyroscope angle random walk errors, for example, might be specified in deg/\(\sqrt{\text{h}}\). Most navigation-grade gyroscopes (including RLG, HRG, IFOG) have angle random-walk errors in the order of \(10^{-3}\) deg/\(\sqrt{\text{h}}\) or less.

Harmonic noise can be caused by temperature control systems, such as building heating, ventilation, and air conditioning (HVAC) systems, which introduce cyclical errors. Also, suspension and structural resonances of the host vehicle can introduce harmonic accelerations, which can excite acceleration-sensitive error sources in gyro sensors.

Noise characterized by a power spectral density that falls off as \(1/f\) with frequency \(f\) is called “1/f noise.” This type of noise is present in most electronic devices, but its causes are not well understood. 1/f noise is usually modeled as some combination of white noise and random walk.

The last type of errors we consider here are deterministic errors. These errors, which can be calibrated and compensated, are sensor output errors in addition to the additive zero-mean white noise and time-correlated noise discussed above. The same models apply to gyroscopes and accelerometers. Some of the common types of sensor errors are illustrated in Figure 1.

Sensor compensation is the process of determining the sensor input from measured sensor output, which is only possible if the sensor input/output relationship is known and invertible. Deadzone errors and quantization errors are associated with this problem. The cumulative effects of both types (deadzone and quantization) often benefit from zero-mean input noise or dithering. Furthermore, not all digitization methods have equal cumulative effects. Output quantization errors are bounded by \(\pm0.5\) least significant bit (LSB) of the digitized output, but the variance of cumulative errors from independent sample-to-sample A/D conversion errors can grow linearly with time.

In developing a gyro sensor error model, it is essential to understand the gyro’s characteristic error sources and behavior. For example, the RLG has a deadzone “lock-in” error due to light scattered from its mirrors; this nonlinearity can be overcome by mechanical or electronic dithering. However, in spite of such individual gyro differences, we can often rely on a general-purpose parametric model for the measured angular rate about the \(x\) axis, such as
\[
\bar{\omega}_x = (S + \delta S)\omega_x + B_{x} + B_{x}\omega_{x} + M_{g}\omega_{y} + M_{g}\omega_{z} + B_{xy}\omega_{y} + B_{xz}\omega_{z} + \eta_{x},
\]
where
\[
\begin{align*}
S & = \text{scale factor (rad/s/V)} \\
\delta S & = \text{scale factor error (rad/s/V)} \\
\omega_{x}, \omega_{y}, \omega_{z} & = \text{true angular rates about the } x, y, z \text{ axes}
\end{align*}
\]

Additional terms for less common error sources include vibration rectification and aniso-inertial. In the latter, spinning gyroes may have slightly unequal moments of inertia about different axes. The resulting biases are proportional to products of angular rates about a pair of axes.

Next, let me discuss some different levels of calibration. Sensor-level calibration can be performed after the gyro is assembled and tested, where calibration coefficients, such as bias and scale factor, can be determined over temperature ranges. Additional real-time calibration can be performed during operation, often by using a Kalman filter to estimate cumulative random gyro errors. In GNSS/INS navigation, for example, GNSS measurements during alignment and navigation can be used to compensate INS sensors [2], [4], [5].

To “spec” the gyro, we need to specify all of the significant gyro errors and their statistical characteristics, as well as application-specific characteristics, such as the dynamic range and bandwidth of the inputs. Gyro-error specifications usually include the stability and turn-on repeatability of gyro bias and scale factor, gyro noise (attitude random walk), input-axis misalignment, and \(g\)-sensitivity. Bias-error repeatability and stability are shown in Figure 2. In some applications we must also consider second-order effects such as \(g^2\)-sensitivity and cross-axis coupling.

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Gyro-error specifications are generally determined during factory testing and calibration. Typical performance grades for gyroscopes are shown in Table 2. These specs are generally accepted for gyros, based on their intended applications, but not necessarily including integrated GNSS/INS applications. Furthermore, these specifications are only rough order-of-magnitude ranges for the various error characteristics. Sensor requirements are largely determined by the application. For example, gyros for gimballed systems generally require much smaller input dynamic ranges and bandwidths than those for strapdown applications.

In many applications, gyro errors such as biases and scale factors in (1), are estimated by using additional sensor data along with a nonlinear Kalman filter. These parameters are fed back to determine the attitude of the vehicle [6], [7].

**AUTHOR INFORMATION**

Mohinder Grewal received the Ph.D. in electrical engineering from the University of Southern California, Los Angeles, in 1974, with a specialization in control systems and computers. He is currently a professor of electrical engineering at California State University, Fullerton, and a consultant in the area of Kalman filtering applications. He has coauthored two books, and has published numerous papers in journals and proceedings.

Angus Andrews received the Ph.D. in mathematics from the University of California, Los Angeles, in 1968. Since his retirement from Rockwell Science Center in 2000, he has consulted with Northrop Grumman and Raytheon on performance analysis of space-based sensing systems. He has coauthored two books and published numerous technical papers.

**REFERENCES**


