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Robust Adaptive Control
w/ applications in flight control*

Dr. Heather Hussain
Guidance, Navigation, Controls, and Autonomy (GNC&A)
The Boeing Company, Tukwila, WA
heather.s.hussain@boeing.com

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Abstract

Heather Hussain

With the advent of each next generation technology, demands for a rapidly reconfigurable control system yielding invariant performance under increasingly unknown or widely varying operating conditions becomes crucial. Adaptive control has long been viewed as one such control method, with implementation on high performance aerial vehicles providing nearly uniform performance across the flight envelope even with limited a priori knowledge on the aircraft's aerodynamic characteristics. This adaptation to parametric uncertainties is achieved through a process of online measurement, evaluation, and compensation through the control input.

While the foundations of robust adaptive control theory were laid in the early 1980's, obtaining quantifiable and practically meaningful robust stability margins for adaptive systems remained an open problem. Successful implementation of adaptive control theory as a viable control solution can only be achieved when global robustness properties, especially with respect to unmodeled dynamics, are well understood. This research proposes a solution to this long standing open problem for a class of linear time-invariant plants, whose states are accessible.

With the use of a modified adaptive update law and sufficient conditions of a frequency-domain criterion, it is shown that the underlying closed-loop system has globally bounded solutions. That is, the overall adaptive system is shown to have analytically computable robustness margins that hold for arbitrary initial conditions.

It is also shown that, with these global properties established, specific conditions can be derived under which the advantage of adaptation over non-adaptive solutions for the control of uncertain systems is made clear. This advantage lies in the fact that parameter adaptation allows learning of the uncertainties whenever the effect of unmodeled dynamics is small, leading to small tracking errors and improved robustness margins.

Presenter

HUSSAIN, HEATHER	Heather Hussain received the B.S. degree and M.S. degree in mechanical engineering from the Rochester Institute of Technology, Rochester, NY, USA, in 2012, and the Sc.D. degree in mechanical engineering at the Massachusetts Institute of Technology (MIT), Cambridge, MA, USA in 2017. Her work experience comprises several internships spanning the aerospace and consumer electronics industries– namely, in Product Design at Apple Inc., as a research Scholar at the Munitions Directorate of the Air Force Research Laboratory, and her work in the design and development of verifiable adaptive flight control systems at The Boeing Company. Ms. Hussain's doctoral research at MIT was sponsored by the Boeing Strategic University Initiative under the direction of Dr. Eugene Lavretsky and Dr. Anuradha Annaswamy. Ms. Hussain joined BR&T's Guidance, Navigation, Control, and Autonomy (GNC&A) group in September 2017. Her research interests lie in adaptive control theory, particularly with applications in aerospace. Ms. Hussain is a member of AIAA and IEEE.

Outline

- **Introduction/Motivating Example**
- **Need for Robust Adaptive Control**
- **Significant Earlier Work**
- **Main Result: Computable Robustness Margins for Adaptive Systems**
  - Main Idea & Key Elements
- **"Tutorial" using Numerical Example**
- **Validation via Simulation Studies**
- **Why Adapt?**
  - Operator Equivalence
  - Persistence of Excitation & Parameter Convergence
- **Summary**
  - Limitations & Future Work

Direct Model Reference Adaptive Control (MRAC)

Introduction

- **Adaptive control theory is a mature control discipline that allows for real-time compensation of uncertainties and changes in system dynamics**

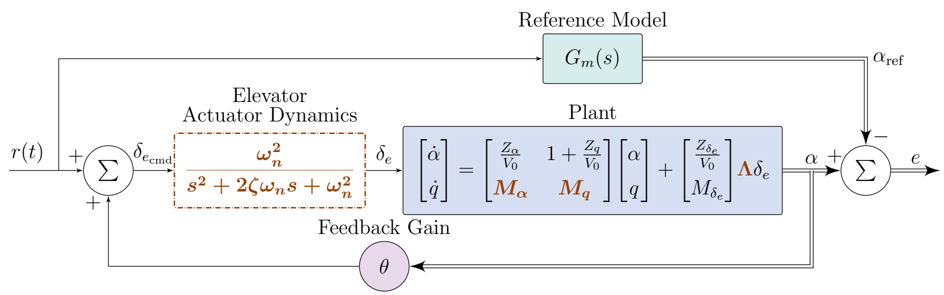
- **Premise: Adapt system parameters to provide a vehicle response that more closely follows the reference model**
  - Graceful degradation in presence of uncertainties
  - Ability to continue mission

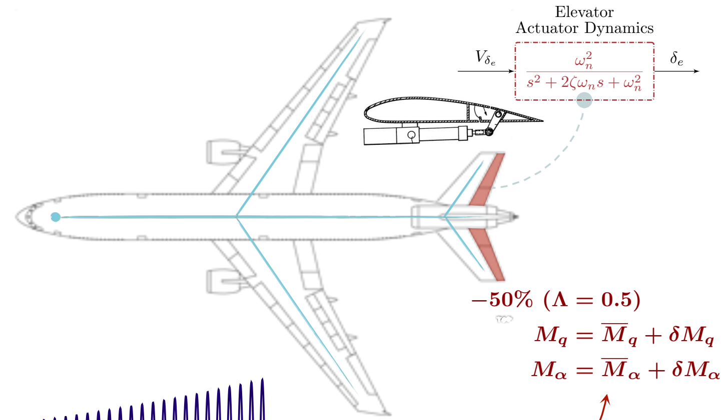
- **Challenge: Gains are bounded nonlinear integral paths → closed loop dynamics are inherently nonlinear**
  - Gain and Phase margin are not defined during adaptation

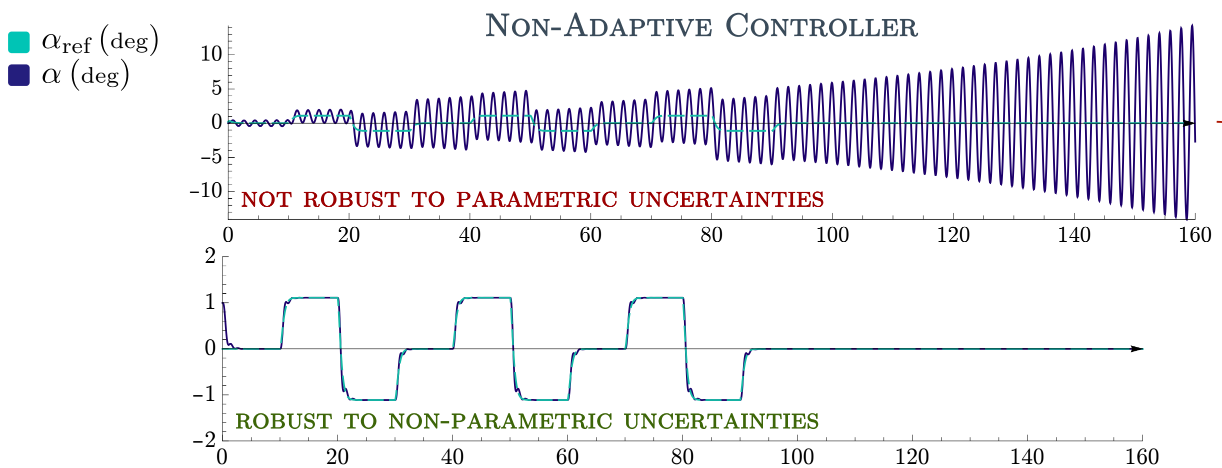
$\frac{Z_q}{V_0} \sim \frac{Z_{\delta_e}}{V_0} \sim \frac{Z_\alpha}{V_0} \sim 0,\ M_\alpha = -\omega_p^2,\ M_q = -2\zeta_p\omega_p,\ M_{\delta_e} = 1,\ (\zeta_p, \omega_p, k_p) = (-1, 1, 1),\ (\zeta_\eta, \omega_\eta, k_\eta) = (0.89, 17, 17^2),\ (\zeta_m, \omega_m, k_m) = (1, 3, 1)$

Motivation

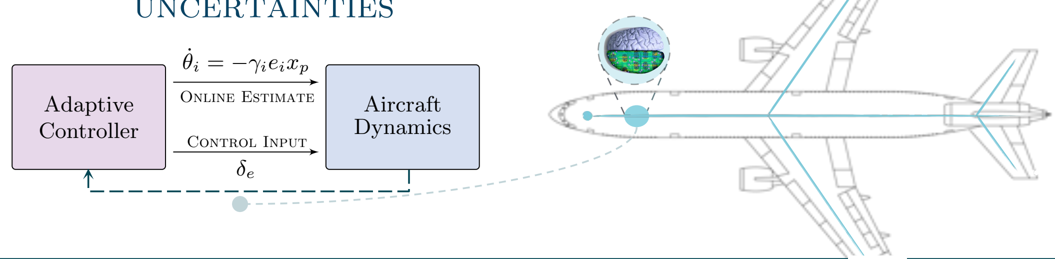
Adaptive Control







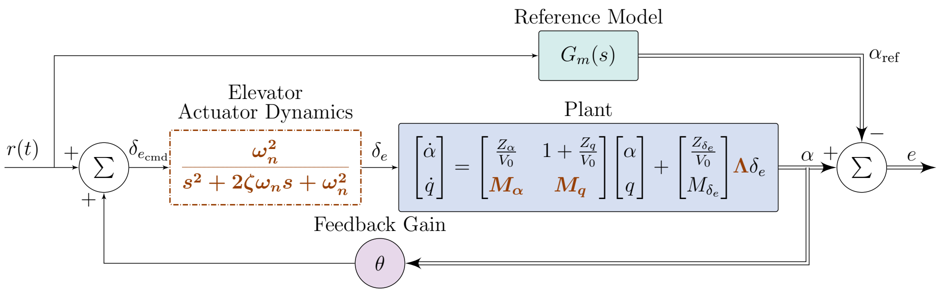
Propose an adaptive controller that improves robustness to parametric uncertainties

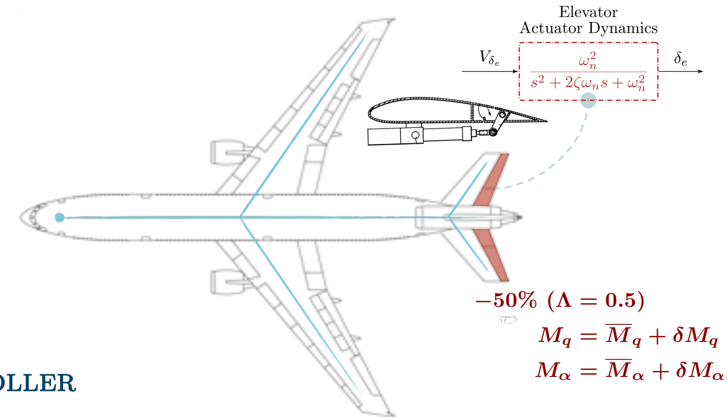


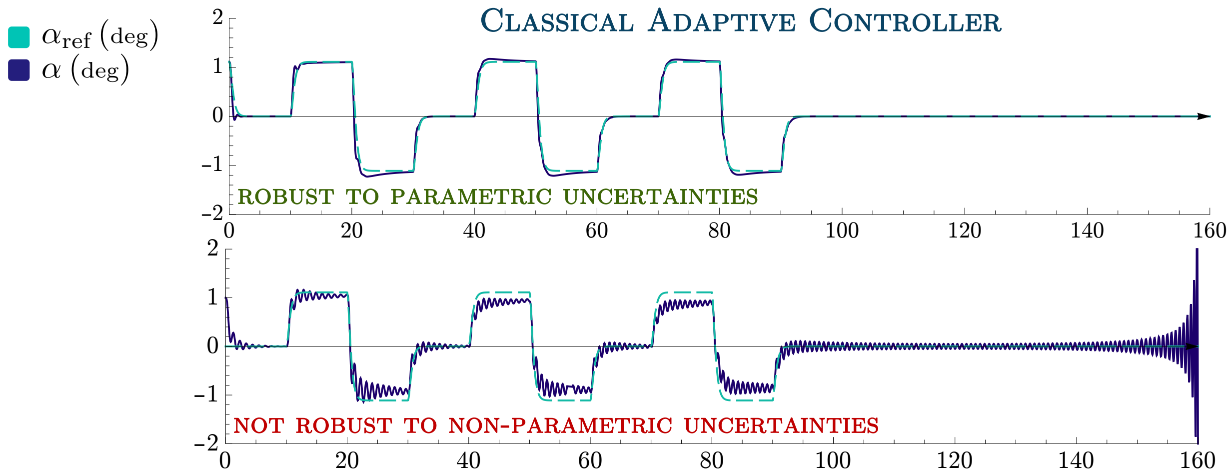
$\frac{Z_q}{V_0} \sim \frac{Z_{\delta_e}}{V_0} \sim \frac{Z_\alpha}{V_0} \sim 0,\ M_\alpha = -\omega_p^2,\ M_q = -2\zeta_p\omega_p,\ M_{\delta_e} = 1,\ (\zeta_p, \omega_p, k_p) = (-1, 1, 1),\ (\zeta_\eta, \omega_\eta, k_\eta) = (0.89, 17, 17^2),\ (\zeta_m, \omega_m, k_m) = (1, 3, 1)$

Motivation

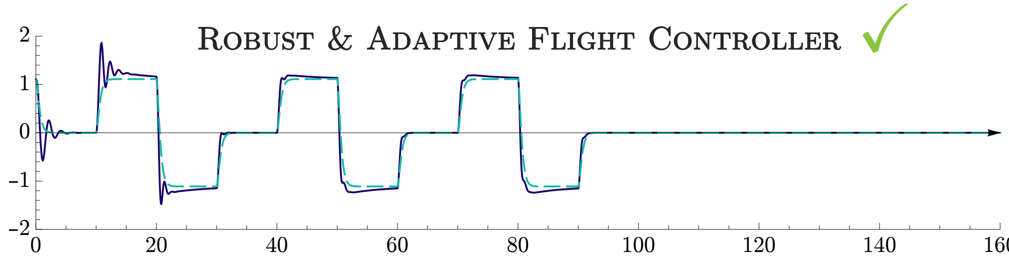
Robust Adaptive Control







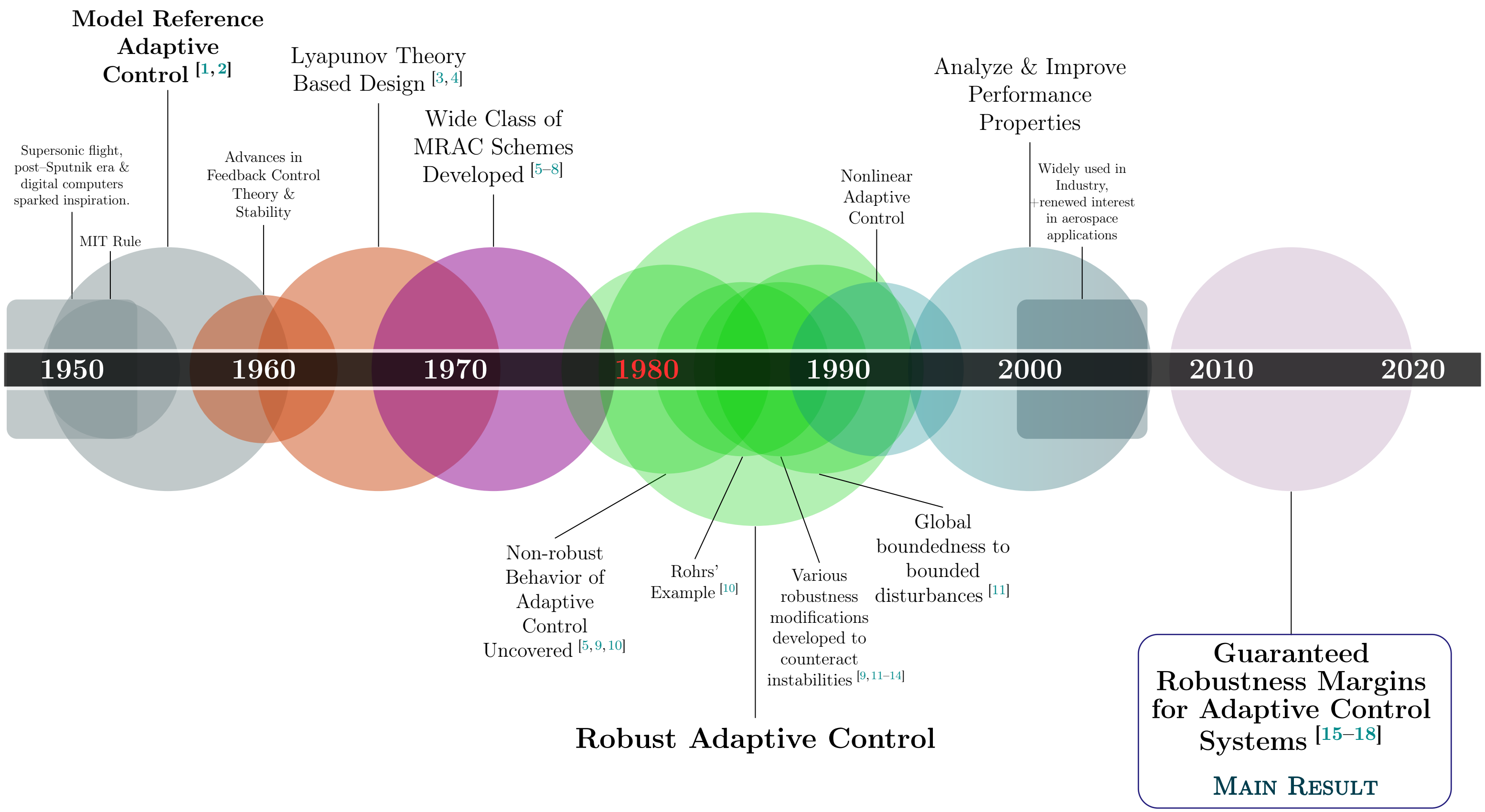
Goal: Find a controller that adapts to **parametric uncertainties** and is robust to **unmodeled dynamics – both are inevitable in real-world systems**



Background

Milestones

"An understanding of fundamental limitations is an essential element in all engineering."
—Gunter Stein, 1989 Bode Lecture

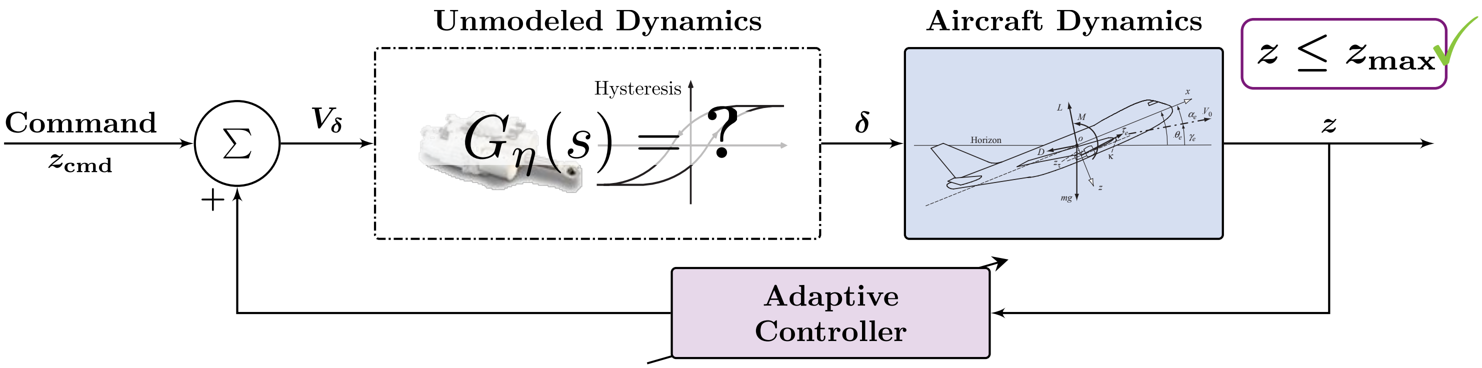


[1] H. P. Whitaker, J. Yamron, A. Kezer, MIT, Design of model-reference adaptive control systems for aircraft. MIT, Instrumentation Laboratory : Jackson & Moreland, 1958. [2] P. V. Osburn, H. P. Whitaker, and A. Kezer, "New developments in the design of model reference adaptive control systems," IAS 29th Annual Meeting, no. Paper 61-39. New York, NY: Institute of the Aerospace Sciences, Jan 1961. [3] B. Shackcloth and R. L. Butchart, Synthesis of Model Reference Adaptive Systems by Liapunov's Second Method. Boston, MA: Springer US, 1965, pp. 145–152. [4] P. Parks, "Liapunov redesign of model reference adaptive control systems," IEEE TAC, vol. 11, no. 3, pp. 362–367, Jul 1966. [5] B. Egardt, Stability of adaptive controllers. New York: Springer Verlag, 1979. [6] K. S. Narendra, Y. H. Lin, and L. S. Valavani, "Stable adaptive controller design - part II: proof of stability," IEEE TAC, vol. 25, pp. 440–448, 1980. [7] I. D. Landau, Adaptive control: the model reference approach. Marcel Dekker, 1979. [8] A. Morse, "Global stability of parameter-adaptive control systems," IEEE TAC, vol. 25, no. 3, pp. 433–439, Jun 1980. [9] P. A. Ioannou and P. V. Kokotovic, Adaptive Systems with Reduced Models. New York: Springer–Verlag, 1983. [10] C.Rohrs, L.Valavani, M.Athans, and G.Stein,"Robustness of continuous-time adaptive control algorithms in the presence of unmodeled dynamics," IEEE TAC, vol. 30, no. 9, pp. 881 – 889, Sep. 1985. [11] K. S. Narendra and A. M. Annaswamy, "Robust adaptive control in the presence of bounded disturbances," IEEE TAC, vol. 31, pp. 306–315, 1986. [12] P. A. Ioannou and P. V. Kokotovic, Adaptive Systems with Reduced Models. Secaucus, NJ, USA: Springer-Verlag New York, Inc., 1983. [13] B. Peterson and K. Narendra, "Bounded error adaptive control," IEEE TAC, vol. 27, no. 6, 1982. [14] S. Naik, P. Kumar, and B. Ydstie, "Robust continuous-time adaptive control by parameter projection," IEEE TAC, vol. 37, no. 2, pp. 182 –197, feb 1992. [15] M. Matsutani, "Robust adaptive flight control systems in the presence of time delay," Ph.D. dissertation, MIT, Feb 2013.

Robust Adaptive Control

Problem Statement

- **Adaptive control needs to be:**
  - **stable** with **parametric** uncertainties
  - **robust** to **non-parametric** uncertainties



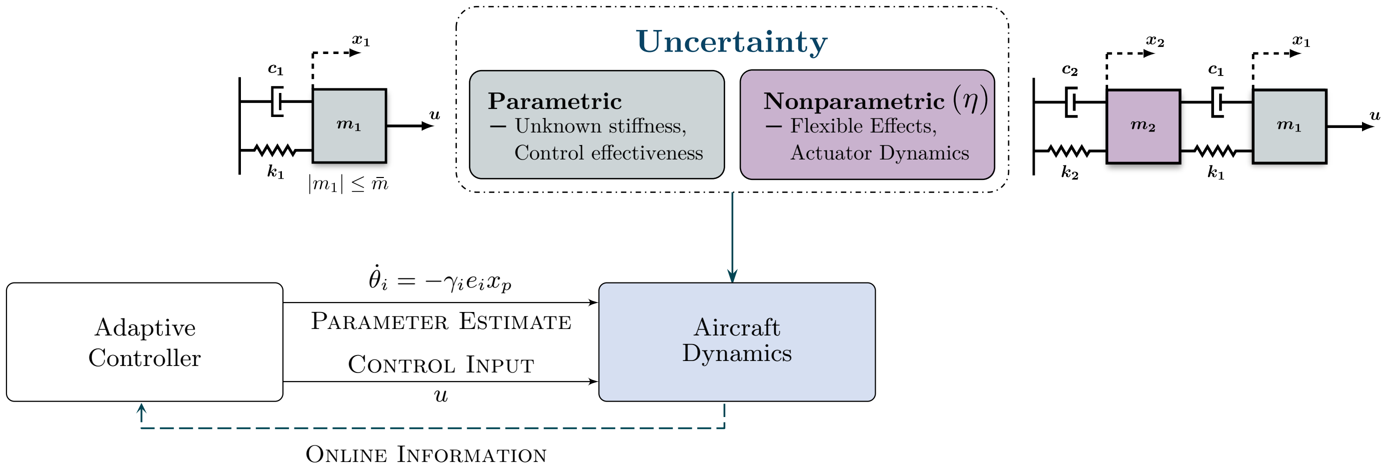
Main Idea: Robustness Margins for Adaptive Systems

- **Significant earlier work[1-4] is conservative.**
  - Global boundedness for a narrow class of unmodeled dynamics[1-3]
  - Semi-global boundedness for a slightly larger class of unmodeled dynamics[3,4]

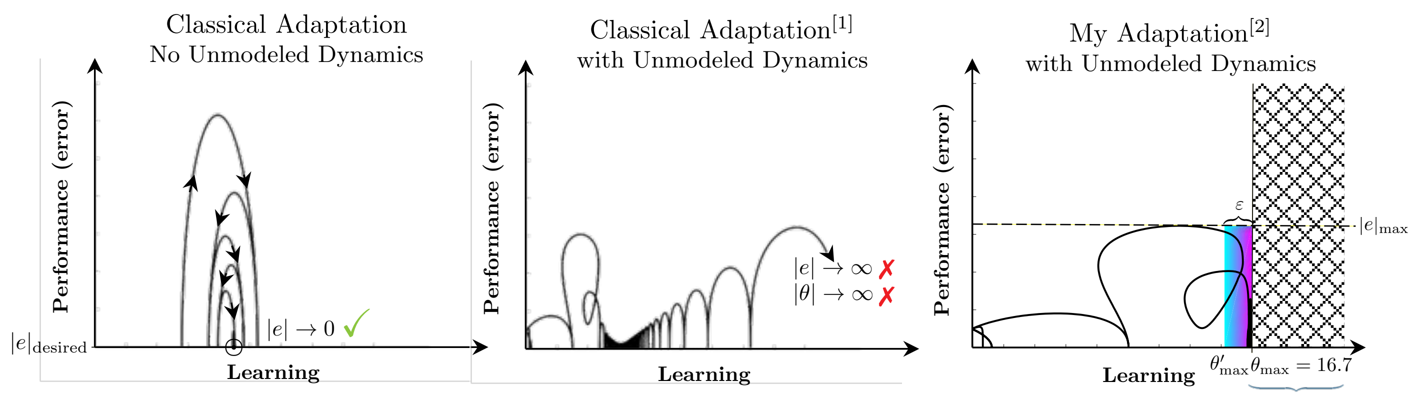
1. S.M. Naik, P.R. Kumar, and B.E. Ydstie. Robust Continuous-time Adaptive Control by Parameter Projection. IEEE Transactions on Automatic Control, Feb 1992.
2. M. Matsutani, A.M. Annaswamy, T. Gibson, and E. Lavretsky. Trustable Autonomous Systems using Adaptive Control. In Proceedings of IEEE Conference on Decision and Control, 2011.
3. P. A. Ioannou and J. Sun. Robust Adaptive Control. Prentice Hall, 1996.
4. K. S. Narendra and A. M. Annaswamy. Stable Adaptive Systems. Prentice Hall, 1989.

Why is this important?

- **Non-parametric uncertainties are inevitable ➔ need robust control solutions**



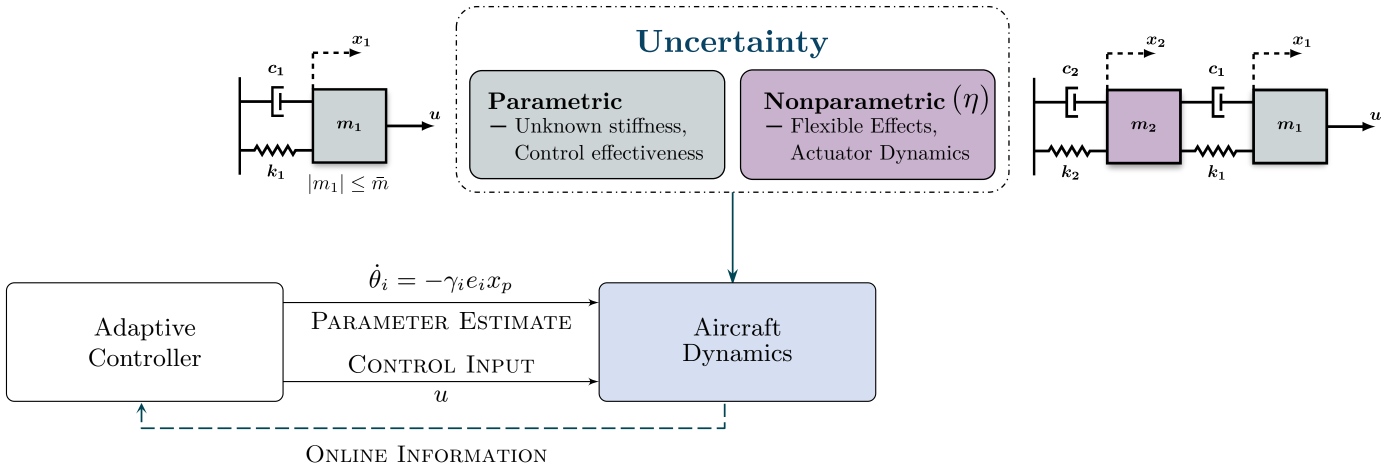
Theoretically verifiable robustness margins



Bounded Learning

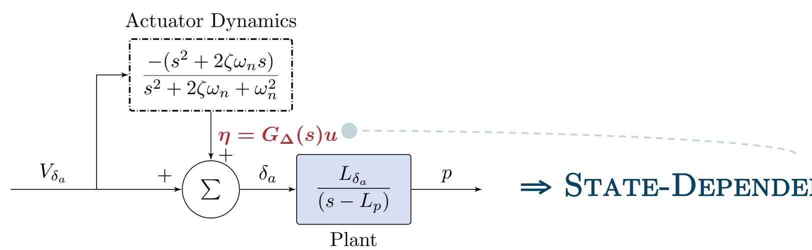
1. C.E. Rohrs, L. Valavani, Athans, M., and G. Stein. Robustness of Continuous-time Adaptive Control Algorithms in the Presence of Unmodeled Dynamics. IEEE TAC Automatic Control, 1985
2. H. Hussain, M. Matsutani, A. Annaswamy, and E. Lavretsky, "A new approach to robust adaptive control", ACC 2016

Why has this been an open problem for so long?



With Parametric Uncertainty: $\theta(t) = \int x_p(\tau) b_m^\top P e(\tau) \mathrm{d}\tau$ $\Rightarrow$ **Nonlinear Time-Varying Closed-Loop**

With Nonparametric Uncertainty:

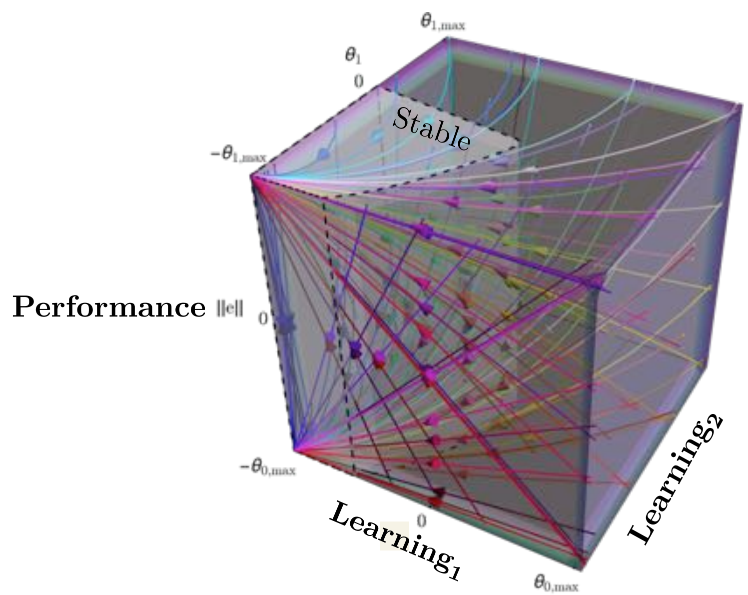


$\Rightarrow$ **State-Dependent Disturbance**

<u>Extremely</u> difficult to show global boundedness

Main Idea

Robustness to Unmodeled Dynamics



Adaptive Law + Projection + Provably Correct Learning Bounds =

> **Main Result:** Analytical Guarantees for global boundedness of projection-based adaptive systems with unmodeled dynamics.

H. Hussain, M. Matsutani, A. Annaswamy, and E. Lavretsky, Adaptive Control of Scalar Plants in the Presence of Unmodeled Dynamics, IFAC ALCOSP, July 2013.
A. Annaswamy, T. Gibson, H. Hussain, and E. Lavretsky. Practical Adaptive Control, 16th Yale Workshop, June 2013.
H. Hussain, A. Annaswamy, and E. Lavretsky, "Robust Adaptive Control in the Presence of Unmodeled Dynamics: A Counter to Rohrs's Counterexample," AIAA Guidance, Navigation, and Control Conference, Aug 2013.
H. Hussain, M. Matsutani, A. Annaswamy, and E. Lavretsky, "Computable Delay Margins for Adaptive Systems with State Variables Accessible," IEEE TAC (To Appear).
H. Hussain, A. Annaswamy, and E. Lavretsky, "A New Approach to Robust Adaptive Control," ACC, July 2016.
H. Hussain, A. Annaswamy, and E. Lavretsky, "Adaptive control of second-order plants in the presence of unmodeled dynamics," IFAC ALCOSP, June 2016.
H. Hussain, C. Sharma-Subedi, A. Annaswamy, and E. Lavretsky, "Robustness of Adaptive Control Systems to Unmodeled Dynamics: A Describing Function Viewpoint," AIAA GNC, January 2017

Main Result

Robustness to Unmodeled Dynamics

Theorem 1(a). *Consider the adaptive system described by*

$$\dot{x} = A_{\rm ol}x + B\Lambda(t)y + b_{\rm ol}r$$
$$y = \mathscr{C}^\top x \qquad .$$
$$G_{\rm ol}(s) \triangleq \mathscr{C}^\top(s\mathbb{I} - A_{\rm ol})^{-1}B$$

If the transfer function matrix

$$Z(s) \triangleq (\mathbb{I} + \Lambda_{\max}G_{\rm ol}(s))(\mathbb{I} - \Lambda_{\max}G_{\rm ol}(s))^{-1}$$

is strictly positive real with

$$\Lambda_{\max} = \mathrm{diag}(\vartheta_{0,\max}, \vartheta_{1,\max}, \cdots, \vartheta_{n-1,\max})$$

then every solution of the closed-loop adaptive system is globally bounded for all $|\vartheta_i(t_0)| \leq \vartheta_{i,\max}$.

Theorem 1(b). *Suppose* $r(t) = 0\ \forall t$.

$$\dot{x} = (A_0 + \tilde{A}(t))x + B_{\mathbb{1}}\Lambda_{\mathbb{1}}(t)y_{\mathbb{1}}$$
$$y_{\mathbb{1}} = \mathscr{C}_{\mathbb{1}}^\top x \qquad .$$
$$G_{\mathbb{1}}(s) \triangleq \mathscr{C}_{\mathbb{1}}^\top(s\mathbb{I} - A_0)^{-1}B_{\mathbb{1}}$$

If the transfer function matrix

$$Z_{\mathbb{1}}(s) \triangleq (\mathbb{I} + \Lambda_{\mathbb{1},\max}G_{\mathbb{1}}(s))(\mathbb{I} - \Lambda_{\mathbb{1},\max}G_{\mathbb{1}}(s))^{-1}$$

is strictly positive real with

$$\Lambda_{\mathbb{1},\max} = \mathrm{diag}(\vartheta_{1,\max}, \cdots, \vartheta_{n-1,\max})$$

then the origin of the adaptive system is globally asymptotically stable for all $|\vartheta_i(t_0)| \leq \vartheta_{i,\max}$.

$A_0 = A_{\rm ol} - \vartheta_{0,\max}p_{bb}^{-1}b_{\rm ol}C_0^\top,\ \tilde{A}(t) = (\vartheta_0(t) + \vartheta_{0,\max})p_{bb}^{-1}b_{\rm ol}C_0^\top.$

Plant	$\dot{\mathbf{x}}_{\mathbf{p}}(t) = A_p\mathbf{x}_{\mathbf{p}}(t) + b_pv(t)$
Unmodeled Dynamics	$\dot{x}_\eta(t) = A_\eta x_\eta(t) + b_\eta u(t),$ $v(t) = {c_\eta}^T x_\eta(t)$ $G_\eta(s) \triangleq c_\eta^\top(sI_{nxn} - A_\eta)^{-1}b_\eta$
Reference Model	$\dot{\mathbf{x}}_{\mathbf{m}}(t) = A_m\mathbf{x}_{\mathbf{m}}(t) + b_mr(t)$
Control Law	$u(t) = \boldsymbol{\theta}^\top(t)\mathbf{x}_{\mathbf{p}}(t) + k_rr(t)$
Adaptive Law	$\dot{\boldsymbol{\theta}}(t) = M^{-1}w$ $w_i(t) = \mathrm{Proj}\left(\{M\boldsymbol{\theta}\}_i, -\{M\Gamma\mathbf{x}_{\mathbf{p}}b_m^\top P\mathbf{e}\}_i\right)$

$$\begin{bmatrix}\dot{x}_p\\ \dot{x}_\eta\end{bmatrix} = \begin{bmatrix}A_p & b_pc_\eta^\top\\ 0 & A_\eta\end{bmatrix}\begin{bmatrix}x_p\\ x_\eta\end{bmatrix} + \begin{bmatrix}b_pd_\eta\\ b_\eta\end{bmatrix}u \implies \begin{aligned}\dot{x} &= A_{\rm ol}x + B\underline{\Lambda(t)y} + b_{\rm ol}r\\ y &= \mathscr{C}^\top x\end{aligned}$$

${c_\eta}^\top \in \mathbb{R}^{1\times m}$, $x_\eta \in \mathbb{R}^{m\times 1}$, and $A_\eta \in \mathbb{R}^{nxn}$, $A_m \in \mathbb{R}^{2\times 2}$ is Hurwitz

$C = [c_0\ c_{\mathbb{1}}]^\top,\ CP^{-1}C^\top = \mathbb{I},\ M = p_{bb}CP^{-1}$ with $b_m \equiv b_p \in \Re^{[n\times 1]},\ A_m^\top P + PA_m < -Q,\ \Gamma = \gamma' P, \gamma' > 0,\ p_{\rm bb} = \sqrt{b_m^\top Pb_m}$

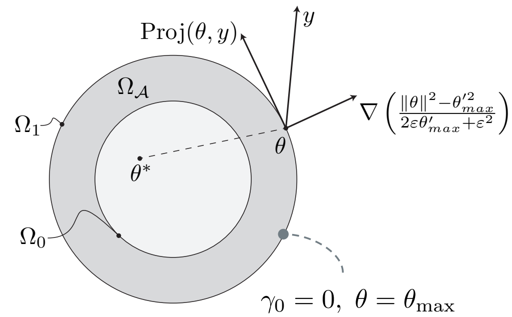
$A_{\rm ol} = \begin{bmatrix}A_p & b_pc_\eta^\top\\ \mathbb{0} & A_\eta\end{bmatrix}, b_{\rm ol} = \begin{bmatrix}b_pd_\eta\\ b_\eta\end{bmatrix}, C_i^\top = \begin{bmatrix}c_i^\top & \mathbb{0}\end{bmatrix}, B = p_{bb}^{-1}\begin{bmatrix}b_{\rm ol} & \cdots & b_{\rm ol}\end{bmatrix}, \mathscr{C}^\top = [C\ \ \mathbb{0}],\ \Lambda = \mathrm{diag}(\vartheta_0, \cdots, \vartheta_{n-1})$

H. Hussain,, "Adaptive Control in the presence of Unmodeled Dynamics," MIT Thesis, June 2017.

Three Key Elements

Projection Modification

$$\dot{\theta}(t) = \gamma \operatorname{Proj}\left(\theta(t), -x_p(t)e(t)\right), \ \gamma > 0$$



Parameter is bounded for all time

Transformation of States

Transformed Error

$$\dot{e} = A_m e + b_p \widetilde{\theta}^\top x_p + b_p \eta \quad \xrightarrow{\ \mathscr{E} \equiv Ce\ } \quad \begin{bmatrix} \dot{\mathscr{E}}_0 \\ \dot{\mathscr{E}}_{\mathbb{1}} \end{bmatrix} = \begin{bmatrix} (\alpha_{00} + \widetilde{\vartheta}_0)\mathscr{E}_0 + (a_1 + \widetilde{\vartheta}_1)\mathscr{E}_1 + p_{bb}\eta \\ \mathscr{A}'_m \mathscr{E}_{\mathbb{1}} + a_0 \mathscr{E}_0 \end{bmatrix}$$

Transformed Parameter

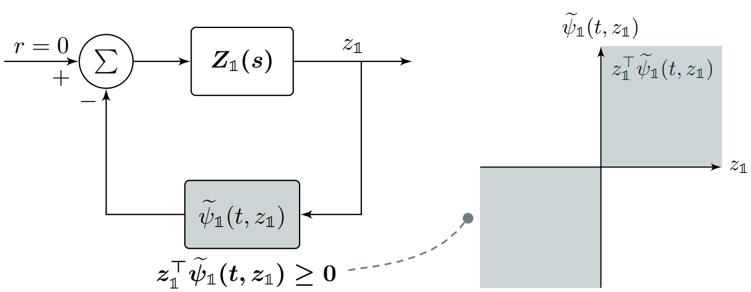
$$\dot{\theta} = M^{-1}\dot{w} \quad \xrightarrow{\ \vartheta \equiv M\theta\ } \quad \dot{\vartheta}_i = \gamma' \operatorname{Proj}(\vartheta_i, -(\mathscr{E}_i + m_i)\mathscr{E}_0)$$

$$\dot{w}_i = \operatorname{Proj}\left(\{M\theta\}_i, -\{M\Gamma x_p b_m^\top Pe\}_i\right) \qquad \triangleq -\gamma_i(\mathscr{E}_i + m_i)\mathscr{E}_0$$

Pertubation is isolated to crucial scalar state

$C = [c_0 \ c_{\mathbb{1}}]^\top, \ CP^{-1}C^\top = \mathbb{I}, \ M = p_{bb}CP^{-1}$ with $b_m \equiv b_p \in \Re^{[n\times 1]}, \ A_m^\top P + PA_m < -Q, \ \Gamma = \gamma' P, \gamma' > 0, \ p_{bb} = \sqrt{b_m^\top P b_m}$

Absolute Stability Framework

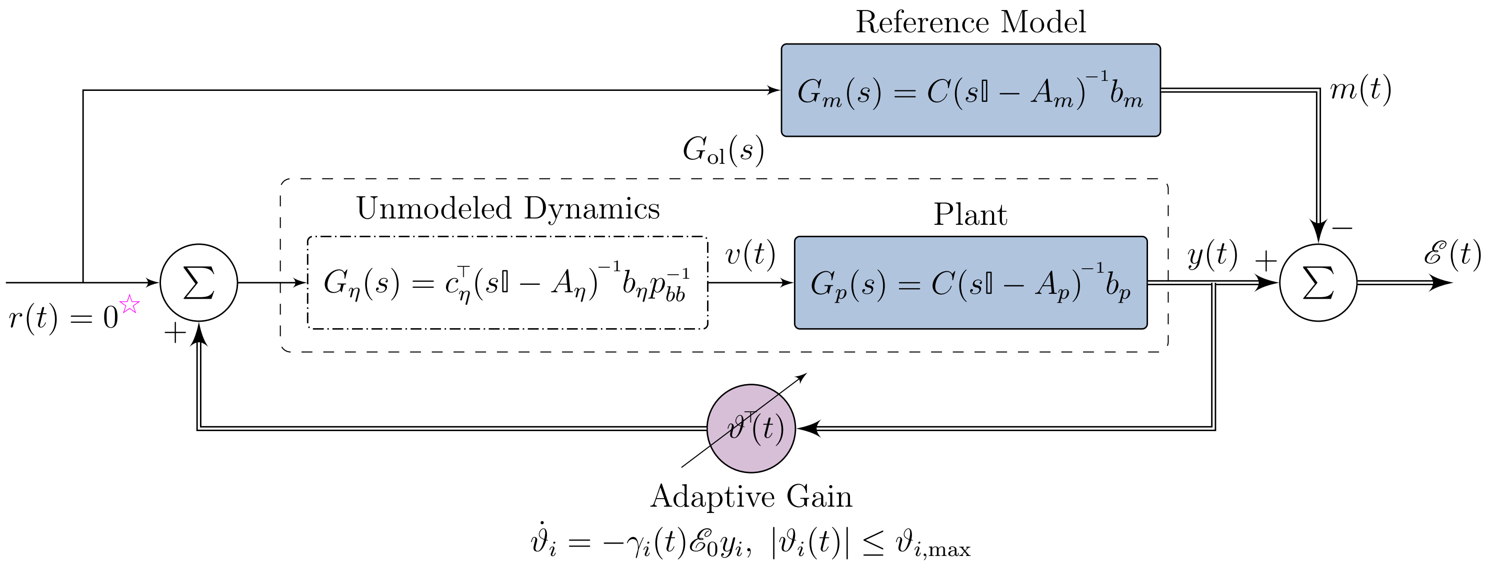


Perform loop transformations, and enforce a stable feedback interconnection between linear subsystem and nonlinearity isolated to feedback path

H. Hussain,, "Adaptive Control in the presence of Unmodeled Dynamics," MIT Thesis, June 2017.

Robust Adaptive Control Problem

Reformulated



$$\dot{\boldsymbol{x}} = \boldsymbol{A}_0 \boldsymbol{x} + \boldsymbol{B}(t)\boldsymbol{y}(t)$$
A_0 Hurwitz
$$\|B(t)\| \leq \bar{B}$$
$$y(t) \in \mathscr{L}_2$$

Adaptive Laws & Input-Output Stability

$\|\widetilde{A}(t)\| > \epsilon$

$$\dot{x} = \left(A_0 + \widetilde{A}(t)\right)x - B_{\mathbb{1}}\psi_{\mathbb{1}}(t, y_{\mathbb{1}})$$
$$y_{\mathbb{1}} = C_{\mathbb{1}}^\top x$$
$$\psi_{\mathbb{1}}(t, y_{\mathbb{1}}) = -\Lambda_{\mathbb{1}}(t) y_{\mathbb{1}}$$
$$G_{\mathbb{1}}(s) \triangleq C_{\mathbb{1}}^\top (s\mathbb{I} - A_0)^{-1} B_{\mathbb{1}}$$

$\|\widetilde{A}(t)\| \leq \epsilon$

$$\dot{\boldsymbol{x}} = (\boldsymbol{A}_0 + \boldsymbol{B}(t))\boldsymbol{x} + \widetilde{A}(t)\boldsymbol{x}$$
A_0 Hurwitz
$$B(t)x \in [-\Lambda_{\max}, \Lambda_{\max}]$$
$$\lim_{t\to\infty} \|\widetilde{A}(t)\| = 0$$

Use Absolute Stability Framework

Closed-Loop Adaptive System

How do I actually apply this?

Second—Order Plant

Theorem 1(b). *Suppose $r(t) = 0\ \forall t$. Consider the adaptive system described by*

$$\begin{aligned} \dot{x} &= (A_0 + \tilde{A}(t))x + B_{\mathbb{1}}\Lambda_{\mathbb{1}}(t)y_{\mathbb{1}} \\ y_{\mathbb{1}} &= \mathscr{C}_{\mathbb{1}}^\top x \\ G_{\mathbb{1}}(s) &\triangleq \mathscr{C}_{\mathbb{1}}^\top (s\mathbb{I} - A_0)^{-1}B_{\mathbb{1}} \end{aligned}.$$

If the transfer function matrix

$$Z_{\mathbb{1}}(s) \triangleq (\mathbb{I} + \Lambda_{\mathbb{1},\max} G_{\mathbb{1}}(s))(\mathbb{I} - \Lambda_{\mathbb{1},\max} G_{\mathbb{1}}(s))^{-1}$$

is strictly positive real with $\Lambda_{\mathbb{1},\max} = \mathrm{diag}(\vartheta_{1,\max}, \cdots, \vartheta_{n-1,\max})$, then the origin of the adaptive system is globally asymptotically stable for all $|\vartheta_i(t_0)| \leq \vartheta_{i,\max}$.

Steps to design a robust adaptive controller

① Using the reference model, compute p_{bb}^{-1} and transformation matrix C

② Assemble closed-loop dynamics and derive $G_{\mathbb{1}}(s)$

$$\left.\left[\begin{array}{cc|c} A_\eta & b_\eta p_{bb}^{-1}\vartheta_0 c_0^\top & b_\eta p_{bb}^{-1} \\ b_p c_\eta^\top & A_p & \mathbb{0} \\ \hline \mathbb{0} & c_1^\top & \mathbb{0} \end{array}\right]\right|_{\vartheta_0 = -\vartheta_{0,\max}} \implies G_{\mathbb{1}}(s)$$

③ Let $Z_{\mathbb{1}}(s) \triangleq (\mathbb{I} + \vartheta_{1,\max} G_{\mathbb{1}}(s))(\mathbb{I} - \vartheta_{1,\max} G_{\mathbb{1}}(s))^{-1}$

④ Find conditions on $(A_\eta, b_\eta, c_\eta^\top)$ or parameter bounds $\vartheta_{i,\max}$ such that $Z_{\mathbb{1}}(s)$ is SPR for all admissible plant parameters.

Easy to satisfy & check

If $G_{\mathbb{1}}(s)$ is Hurwitz, $\alpha \leq \vartheta_1(t) \leq \beta$, with $\alpha < 0 < \beta$ and the Nyquist plot of $G_{\mathbb{1}}(s)$ lies in the interior of the disk $\mathscr{D}(\alpha, \beta)$ then $Z_{\mathbb{1}}(s)$ is SPR and Theorem 1(b) holds.

$C = [c_0\ c_{\mathbb{1}}]^\top$, $CP^{-1}C^\top = \mathbb{I}$, $M = p_{bb}CP^{-1}$ with $b_m \equiv b_p \in \Re^{[n\times 1]}$, $A_m^\top P + PA_m < -Q$, $\Gamma = \gamma' P, \gamma' > 0$, $p_{bb} = \sqrt{b_m^\top P b_m}$

H. Hussain,, "Adaptive Control in the presence of Unmodeled Dynamics," MIT Thesis, June 2017.

$(\zeta_p, \omega_p, k_p) = (1, 0.133, 0.16)$
$(\zeta_m, \omega_m, k_m) = (1, 0.4, 0.16)$
$(\vartheta_{0,\max}, \vartheta_{1,\max}) = (6, 1.4)$

Numerical Example

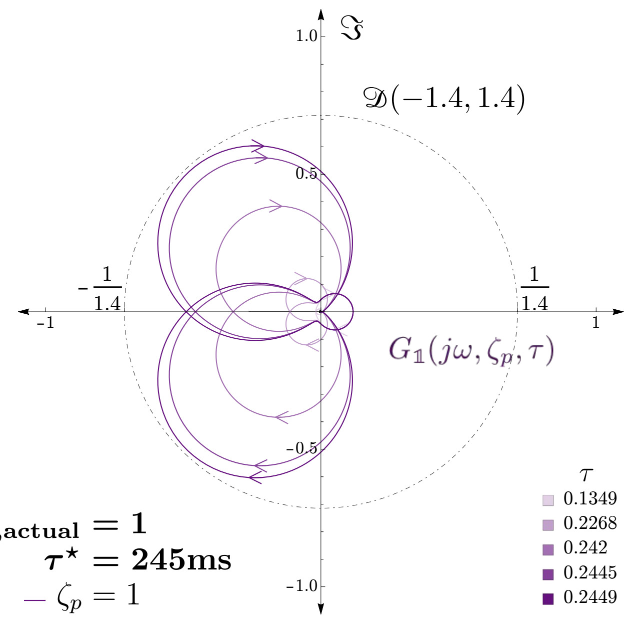
Time Delay

- **Demonstrate applicability using numerical example from previous work** ($\tau^\star_{\mathrm{MM}} = 4\mathrm{ms}$)

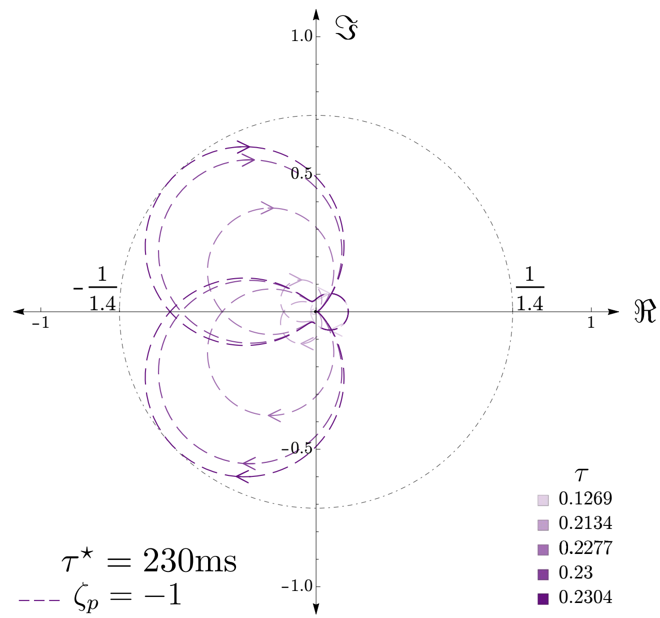
[MM] H. S. Hussain, Y. Yildiz, M. Matsutani, A. M. Annaswamy, and E. Lavretsky, "Computable delay margins for adaptive systems with state variables accessible," IEEE Transactions on Automatic Control, vol. PP, no. 99, pp. 1–1, 2017.

- **2nd Order Plant in the presence of an input time delay –Use 2nd Order Padé approximation**
- **Plant contains parametric uncertainty and can be stable or unstable**

$$G_{\mathbb{1}}(s) = \frac{\left(\frac{\omega_m\sqrt{\omega_m^4+6\omega_m^2+1}}{\omega_m^2+1}\right)\left(s^2 - \frac{6s}{\tau} + \frac{12}{\tau^2}\right)}{\left(s+2\zeta_p\omega_p s+\omega_p^2\right)\left(s^2+\frac{6s}{\tau}+\frac{12}{\tau^2}\right)+\left(s+\frac{2\omega_m}{\omega_m^2+1}\right)\left(s^2-\frac{6s}{\tau}+\frac{12}{\tau^2}\right)\vartheta_{0,\max}}.$$



$\zeta_{p,\mathrm{actual}} = 1$
$\tau^\star = 245\mathrm{ms}$
— $\zeta_p = 1$



$\tau^\star = 230\mathrm{ms}$
--- $\zeta_p = -1$

Sufficient frequency domain criterion can be checked graphically for lower order systems → Delay margin can be derived using Nyquist Plot & Circle Criterion as

$$\bar{\tau}^\star = \min_{\zeta_p}\left(\max\left(\{\tau \mid G_{\mathbb{1}}(j\omega, \zeta_p, \tau) \in \mathscr{D}(-\vartheta_{1,\max}, \vartheta_{1,\max})\}\right)\right)$$

H. Hussain,, "Adaptive Control in the presence of Unmodeled Dynamics," MIT Thesis, June 2017.

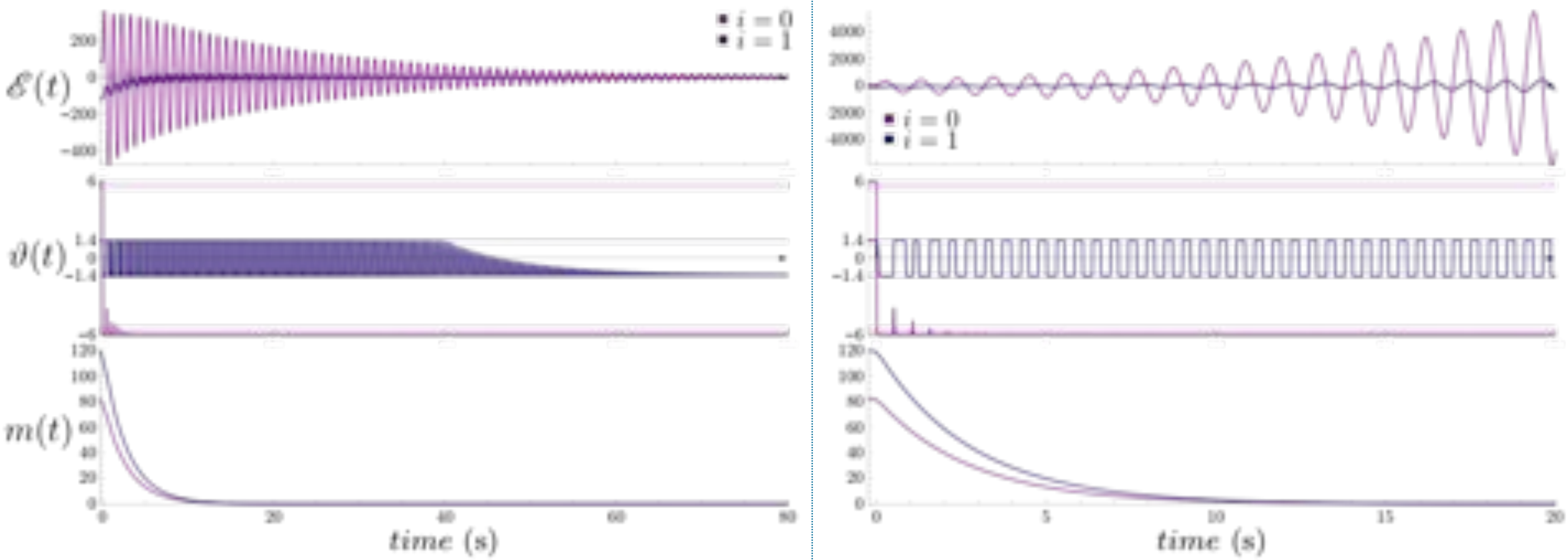
Simulation Studies

Time Delay

$\zeta_{p,\text{actual}} = 1$
$\tau^\star = 245\text{ms}$

- Improved analytical delay margin from $\tau^\star_{\text{MM}} = 4\text{ms}$ to $\tau^\star = 245\text{ms}$
- Simulation studies validate theoretical derivations from Time Delay numerical example

$\tau < \tau^\star \implies$ Asymptotic Stability | $\tau \geq \tau^\star \implies$ Instability



(a) $\tau = 241.7$ ms (b) $\tau = 245$ ms

Methodology & approach extends to the case of real (not approximated) time—delay and provides a **PRACTICAL** and **ANALYTICALLY COMPUTABLE DELAY MARGIN**

H. Hussain,, "Adaptive Control in the presence of Unmodeled Dynamics," MIT Thesis, June 2017.

Why Adapt?

Parameter Convergence

Reference Model	$\dot{x}_m(t) = A_m x_m(t) + br(t)$
Plant	$\dot{x}_p(t) = A_p x_p(t) + bu(t)$
Control Input	$u(t) = \theta(t)^\top x_p(t) + r(t)$
Closed-loop	$\dot{x}_p = (A_p + b\theta^\top(t))x_p + br(t)$

Goal: $x_p(t) \to x_m(t) \impliedby \theta(t) \equiv \theta^\star$

$$\underbrace{\boldsymbol{A_p + b\theta^\star = A_m}}_{\textbf{matching condition (1)}}$$

Theorem (Parameter Convergence [Kokotovic et al., 1985]**).** *Consider the system* $\boldsymbol{(\theta(t) \to \theta^\star)}$

$$\begin{bmatrix} \dot{e} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} A & b\omega(t)^\top \\ -\mu\omega(t)h^\top & \mathbb{0} \end{bmatrix} \begin{bmatrix} e \\ \phi \end{bmatrix}.$$

with $\phi \triangleq \theta - \theta^\star$ *and* $\overline{W}_m(s) \triangleq h^\top(s\mathbb{I} - A)^{-1}b$. *Let* $\omega(t)$ *be bounded, almost periodic, and persistently exciting. Then there exists a* $\mu^\star > 0$ *such that for all* $\mu \in (0, \mu^\star]$, *the origin of the system is exponentially stable if*

$$\min_i \Re\left[\lambda_i\left(\int_0^T \omega(t)\overline{W}_m(s)\omega(t)^\top d\tau\right)\right] > 0.$$

$+$ **Unmodeled Dynamics:** $\begin{bmatrix} \dot{\boldsymbol{x}}_p \\ \dot{\boldsymbol{x}}_\eta \end{bmatrix} = \begin{bmatrix} \boldsymbol{A_p} & \boldsymbol{bc_\eta^\top} \\ \boldsymbol{b_\eta\theta^{\star\top}} & \boldsymbol{A_\eta} \end{bmatrix} \begin{bmatrix} \boldsymbol{x_p} \\ \boldsymbol{x_\eta} \end{bmatrix} + \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{b_\eta} \end{bmatrix} \boldsymbol{r(t)} \Big\}$ $\boldsymbol{\nexists\theta^\star}$ **s.t. (1) is satisfied**

Condition 1. *If* $\exists\overline{\theta}$ *such that the equality*

$$\left(\mathbb{I} - G_p(s)G_\eta(s)\overline{\theta}^\top\right)^{-1} G_p(s)G_\eta(s)r(t) = G_m(s)r(t)$$

is satisfied, then an operator matching condition is said to exist for $\overline{\theta}$. *Furthermore, if the adaptive gain* $\theta(t) = \overline{\theta}$, *then the tracking error* $e = x_p - x_m$ *is equal to zero.*

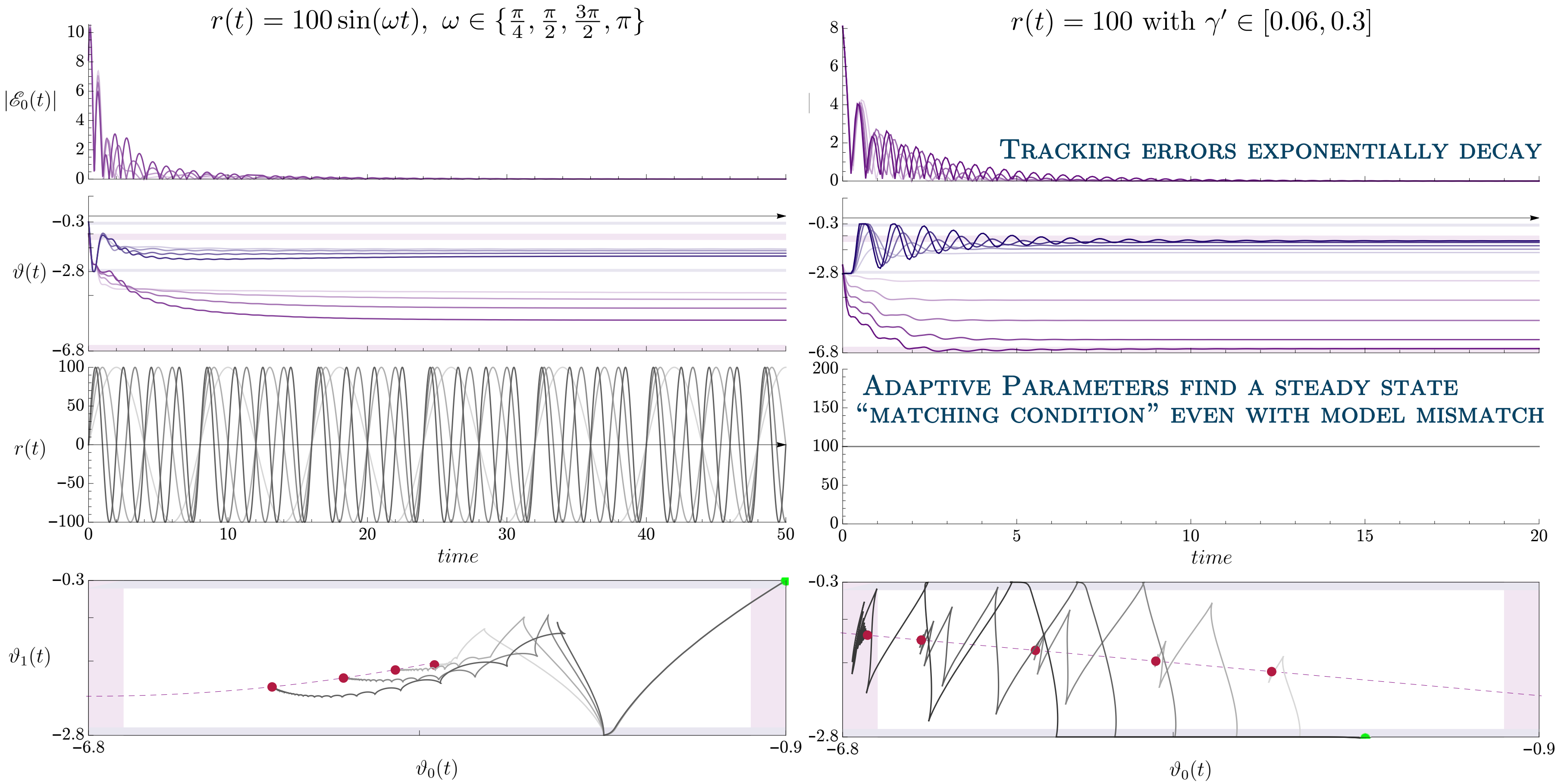
Derived notionally equivalent matching condition, in frequency domain, for which local convergence Theorem holds & original control goal of tracking is achieved.

H. Hussain,, "Adaptive Control in the presence of Unmodeled Dynamics," MIT Thesis, June 2017.

$(\zeta_p, \omega_p, k_p) = (1, 1, 1)$
$(\zeta_m, \omega_m, k_m) = (1, 3, 1)$

Simulation Studies

Parameter Convergence (Rohrs' Unmodeled Dynamics)



Adaptive Controller is able to asymptotically <u>track</u> even in the presence of both parametric and nonparametric uncertainties, due to the existence of an operator matching condition & parameter convergence – **<u>clearly demonstrates the benefit of adaptation over non-adaptive control designs</u>**

H. Hussain,, "Adaptive Control in the presence of Unmodeled Dynamics," MIT Thesis, June 2017.

Summary

Robust Adaptive Control

- **Solved an open problem:**
  - Rigorously proved global boundedness of a closed-loop adaptive system comprised of a LTI nth–order plant, whose state variables are accessible, in the presence of unmodeled dynamics
  - Class of unmodeled dynamics for which the system is robust to is shown to be analytically computable.
- **Reformulated the robust adaptive control problem into a well-known stability framework**
  - Employed the Circle Criterion to analyze stability of the solutions and proved global boundedness
- **Sufficient frequency domain criterion guarantees global boundedness**
- **Verified fundamental trade-off between adaptation & robustness**
- **Validated analytical results via simulation**
  - Demonstrated applicability and practicality of the result
- **Extends to Multiple-Input systems and systems with time-delay**
- **Proved that tracking is still possible even in the presence of unmodeled dynamics, due to the existence of a novel operator matching condition**

- **Limitations & Future Work:**
  - Currently applies to LTI plants whose states are accessible, apply methodology to output feedback
  - Consider other classes of non-parameteric uncertainties and/or nonlinear plants

We have showed the extension of this result to such systems at Boeing.

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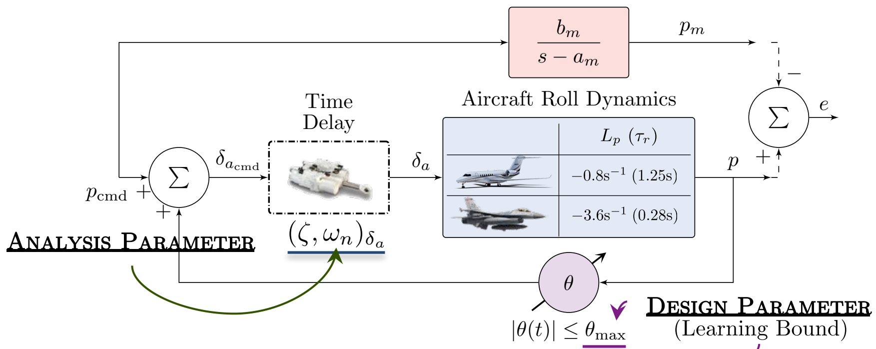
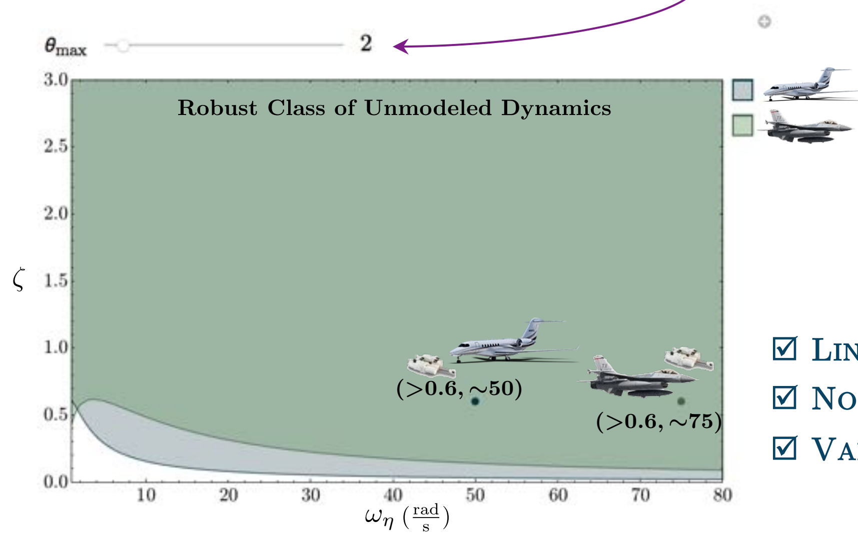
Fundamental Trade-off

Scalar Numerical Example (Roll Dynamics)

- **Adaptive system is robust to all unmodeled dynamics shown in shaded regions (for fast roll dynamics and for slower) given adaptation bound $\theta_{\max}$**





☑ Linear Robustness Margins

☑ Not Conservative

☑ Validated in Simulation

Larger adaptation bound ➔ smaller robust class of unmodeled dynamics